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Introduction to Panel Data Analysis

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Swiss Centre of Expertise in the Social Sciences (FORS)
c/o University of Lausanne

Lugano Summer School, 2016

Introduction panel data, data management

1 Introducing panel data (OL)

2 The SHP (UK)

3 Data Management with Stata (UK)

Regressions with panel data: basic

4 Regression refresher (UK)

5 Causality (OL)

6 Fixed effects models (OL)

7 Random Effects (random intercept) models (OL)

8 Nonlinear regression (UK)

Additional topics

9 Missing data (OL), 10 Random Slope models (OL)

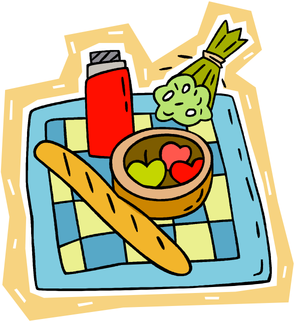
11 Dynamic models (UK)

Organisation

Morning (8.30-12.30, break ca 10.30-10.45)

- Classroom
- Theory and application examples

Lunch Break 12.30-13.30



Afternoon (13.30-17.00, break ca. 15.30-15.45)

Hands-on; aim: apply what we discussed in the morning

– Data management and descriptive analysis with panel data

– Regression models with panel data

Prepared data sets and exercises or work with your own data

Discussion of individual questions whenever possible

Purpose of this Summer School

To introduce basic methods of panel data analysis:

- Emphasis on **causal effects** (within variation) but also **descriptive** methods (OLS)
- **Less emphasis on complex methods** (dynamic models, instruments)
- Practical implementation with **Stata**, do-files (and data)
- Data preparation

- Presenting and **interpreting** results
- **Graphical** display of regression results

Surveys over time: repeated cross-sections vs. panels

- **Cross-Sectional Survey**: conducted at one or several points in time ("*rounds*") *using **different respondents** in each round*

- **Panel Survey**: conducted at several points in time ("*waves*") using ***the same sample respondents over waves***

  → panel *data* mostly from prospective (panel) surveys

  → also: from retrospective ("biographical") survey

Panel Surveys: to distinguish

Length and sample size:

- **Time Series**: N small (mostly=1), T large ($T \to \infty$)

  → time series models (finance, macro-economics, demography, …)

- **Panel Surveys**: N large, T small ($N \to \infty$)

  → social science panel surveys (sociology, microeconomics, …)

Sample

- **General population**:
  - rotating: only few (pre-defined number) waves per individual (in CH: SILC, LFS)
  - indefinitely long (in CH: SHP)
- **Special population**:
  - e.g., age/birth cohorts (in CH e.g.: TREE, SHARE, COCON) representative for population of special age group / birth years

Panel surveys increasingly important

Changing focus in social sciences

→ Life course research: **individual trajectories** (e.g., growth curves, transitions into and out of states)

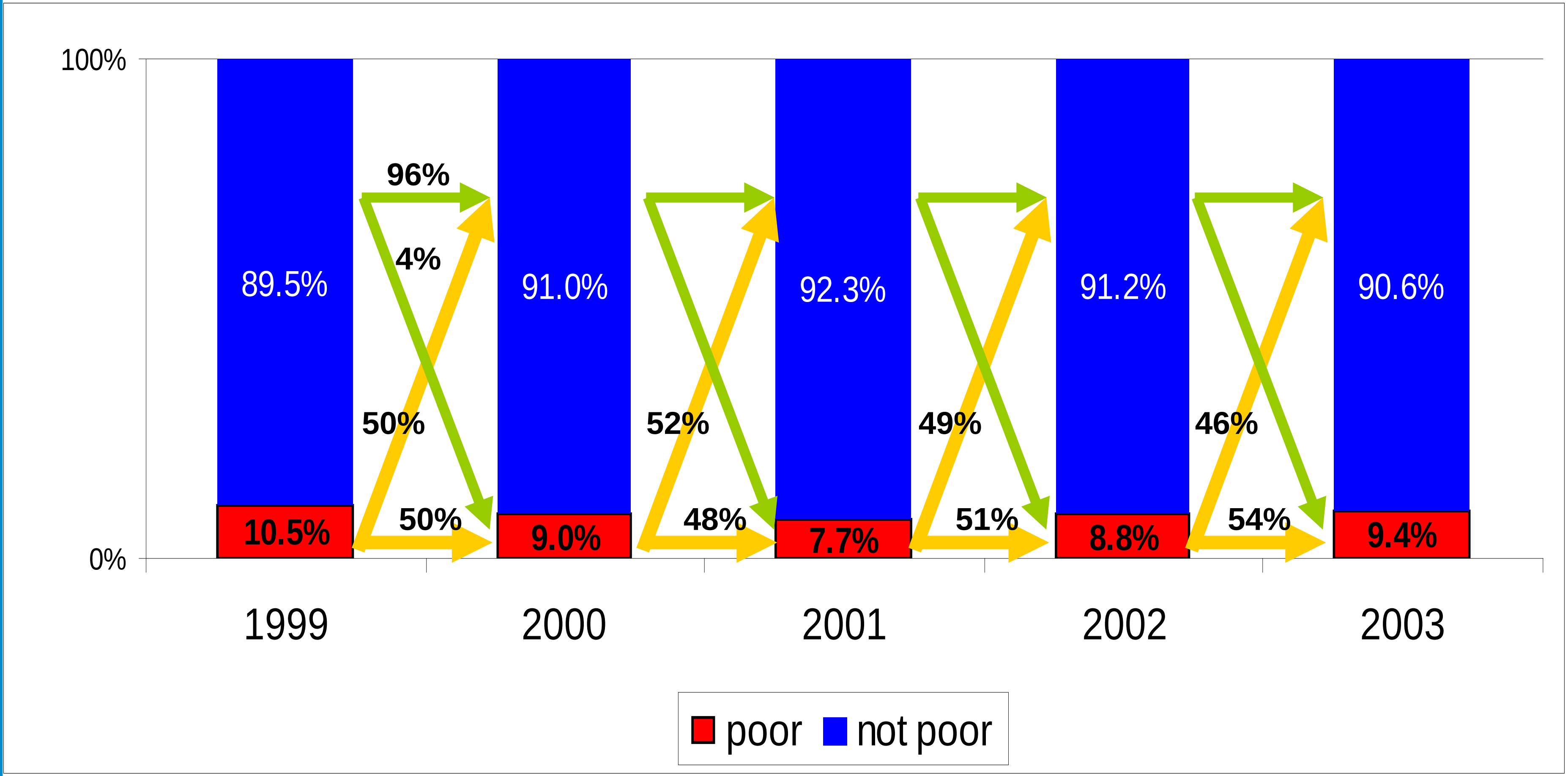
→ Identify “**causal effects**” (unbiased estimates) rather than correlations

→ Large investments in social science panel surveys, high **data quality**!

Analysis potential of panel data

- close to **experimental design**: *before and after* studies of treated
- Control of ***unobserved time-invariant*** *individual characteristics* (FE Models)

Example: Transitions into and out of states (poverty in SHP)



-> **individual dynamics** can only be measured with panel data!

Identification of age, time, and (birth) cohort effects

Fundamental relationship: $a_{it} = t - c_i$ (eg 30 = 2014 - 1984)

- Effects from "formative" years (childhood, youth) -> **cohort effect** (e.g. taste in music)
- Time may affect behavior -> **time effect** (e.g. computer performance, economic cycle)
- Behavior may change over the life cycle-> **age effect** (e.g. health)

- In a ***cross-section*****,** t is constant

  → **age and cohort collinear** (only joint effect estimable)
- In a ***cohort*** **study,** cohort is constant

  → **age and time collinear** (only joint effect estimable)
- In a ***panel*****,** A_{it}, t, and c_i collinear.

  → only two of the three effects can be estimated

  → we can use (t,c_i), (A_{it},c_i), or (A_{it},t), but not all three

Problems of panel surveys

Fieldwork / data quality related

- **High costs** (panel care, tracking households, incentives):

  → increasing number of online panel surveys (randomly selected) e.g., LISS Panel, GiP, GESIS panel, ELIPSS, UK – GenPopWeb initiative)

- **Initial nonresponse** (wave 1) and **attrition** (=drop-out of panel after wave 1):

  → increasing efforts (sampling frame in CH, incentives, tracking, questionnaire modularization, …)

- Panel **conditioning** effects (details largely unknown)

- …

- Finally: you design a panel for the next generation …

2

Introducing the Swiss Household Panel (SHP)

Swiss Household Panel: overview

- Primary goal: **observe social change and changing life conditions** in Switzerland

- First wave in **1999**, more than 5,000 households. Refreshment samples in **2004**, more than 2,500 households, several new questions, and in **2013** (more than 4,500 households, full questionnaire from 2014 on (2013: biographical questionnaire)

- Run by **FORS** (Swiss Centre of Expertise in the Social Sciences), c/o University of Lausanne

- Financed by Swiss National Science Foundation

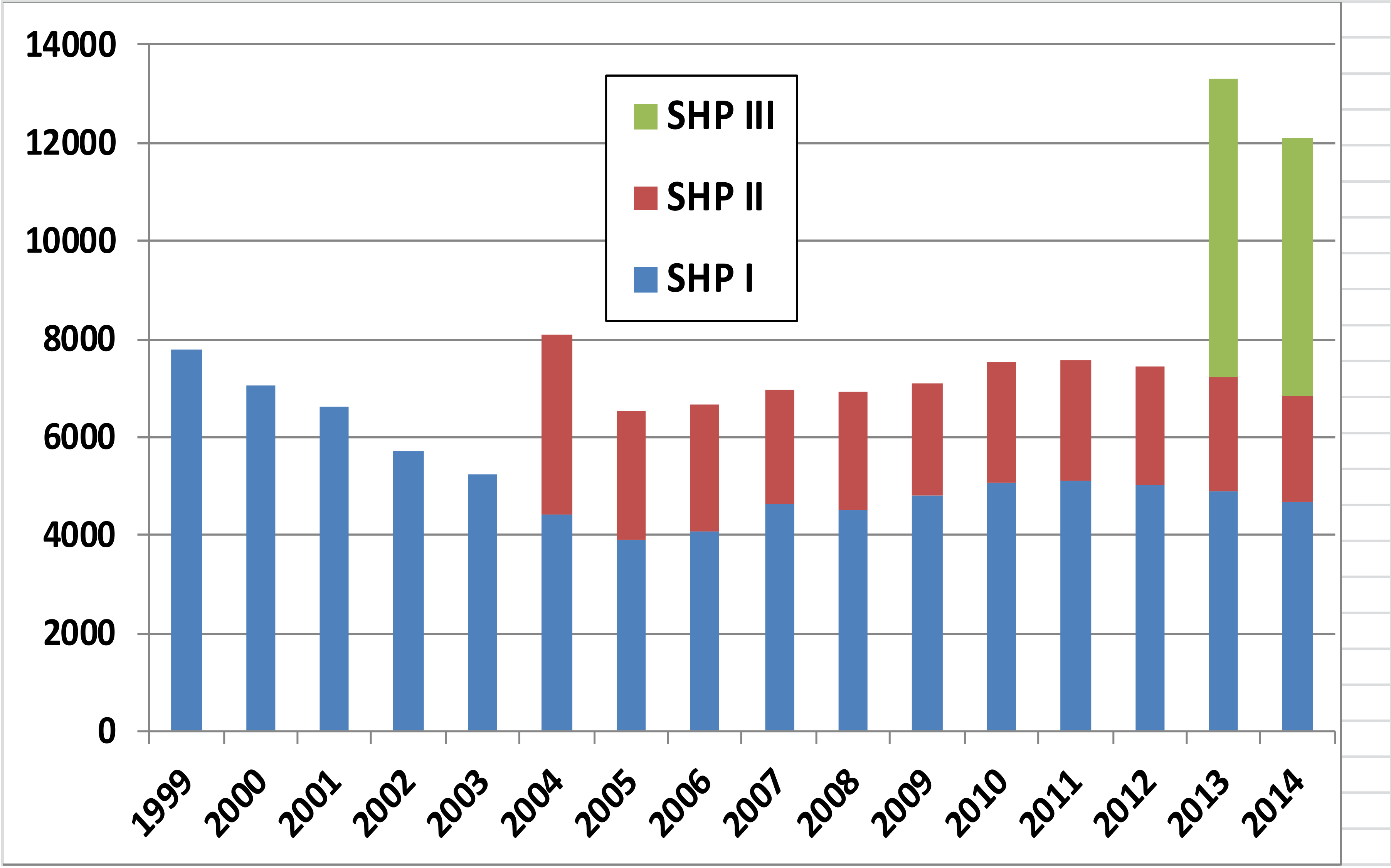
SHP – sample and methods

- Representative of the Swiss residential population
- Each individual surveyed **every year** (Sept.-Jan.)
- All household **members from 14 years on** surveyed (**proxy** questionnaire if child or unable)
- **Telephone interviews** (central CATI), languages D/F/I
- **Metadata**: biography, interviewers, call data

Following rules:

- OSM **followed** if moving, from 2007 on all individuals
- All new **household entrants** surveyed

SHP – sample size (individuals) and attrition



SHP: Survey process and questionnaires

Grid Questionnaire: Inventory and characteristics of hh-members

Persons 18+ years « reference person»	Persons 14+ years	Persons 13- years + « unable to respond»

Household Questionnaire:
housing, finances, family roles, …

Individual Questionnaires:
work, income, health, politics, leisure, satisfaction of life …

Individual Proxy Questionnaires:
school, work, income, health, …

SHP: Questionnaire Content

- **Social structure**: socio-demography, socio-ecomomy, work, education, social origin, income, housing, religion
- **Life events**: marriages, births, deaths, deceases, accidents, conflicts with close persons, etc.
- **Politics :** attitudes, participation, party preference)
- **Social participation**: culture, social network, leisure
- **Perception and values**: trust, confidence, gender
- **Satisfaction**: different satisfaction issues
- **Health:** physical and mental health self-evaluation, chronic problems
- **Psychological scales**

SHP Questionnaire: Rotation modules

Module	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Social network	X			X			X			X	
Religion			X			X			X		
Social participation		X			X			X			X
Politics		X			X			X			X
Leisure	X			X			X			X	
Psychologi-cal Scales			X			X			X		

International Context

SHP is part of the **Cross National Equivalent File** (CNEF) = General population panel survey with data from:

- **USA** (PSID, data since 1980)
- **D** (SOEP, data since 1984)
- **UK** (BHPS, data since 1991, from 2009 Understanding Society)
- **Canada** (SLID, data since 1993)
- **CH** (SHP, data since 1999)
- **Australia** (HILDA, data since 2001)
- **Korea** (KLIPS, data since 1998)
- **Russia** (RLMS-HSE, data since 1995)

More countries will join (South Africa, Israel, Morocco …)

- **Subset** of variables (variables from original files can be added)
- Variables **ex-post harmonized**, names, categories

Frick, Jenkins, Lillard, Lipps and Wooden (2007): “The Cross-National Equivalent File (CNEF) and its member country household panel studies.“ *Journal of Applied Social Science Studies* (Schmollers Jahrbuch)

SHP – structure of the data

- 2 yearly files (currently available: 1999-2014 (+beta 2015))
  - **household**
  - **Individual**

- 5 unique files
  - **master person** (mp)
  - **master household** (mh)
  - **social origin** (so)
  - **last job** (lj)
  - **activity (employment) calendar** (ca)

- Complementary files
  - **biographical** questionnaire (2001/2002, and 2012/2013)
  - **Interviewer data** (2000, and yearly since 2003)
  - **Call data** (since 2005)
  - **CNEF** SHP data variables
  - **Imputed income** variables

Documentation (Website: D/E/F)

forscenter.ch/en/our-surveys/swiss-household-panel/ then link Documentation/FAQ:

- **Questionnaires PDF**
- **User Guide PDF**
- **Variable by Domain** (variable search by topic)
- **List of Variables** (if variable name is known)

- …

SHP – data delivery

- **Data** ready about **1 year after end of fieldwork** – **downloadable** from SHP-server:

  forscenter.ch/en/our-surveys/swiss-household-panel/datasupport-2/telecharger-les-donnees/

  Signed contract with FORS
- Upon contract receipt, **login and password** sent by e-mail
- Data **free of charge**
- Users become **member of SHP scientific network** and **document** all **publications** based on SHP data
- Data **upon request**:
  - Imputed income
  - Call data
  - Interviewer matching ID
  - Context data (special contract); data is matched at FORS

3
Stata and panel data

Why Stata?

Capabilities
- Data management
- Broad range of statistics
  - **Powerful for panel data!**
  - Many commands ready for analysis
  - User-written extensions

Beginners and experienced users
- Beginners: analysis through menus (point and click)
- Advanced users: good programmable capacities

Basics

- Look at the data, check variables
- Descriptive statistics
- Regression analysis

→ **Handout Stata basics**

Working with panel data

- Merge
- Creating « long files »
- Working with the long file
- Add information from other household members

→ **Handout Stata SHP data management (includes Syntax examples, exercises)**

1. Merge: _merge variable

Master file

idpers	p07c44
1	8
2	7
3	9
4	10

using file

idpers	p08c44
3	9
4	5
5	10
6	3

idpers	p07c44	p08c44	_merge
1	8	.	1
2	7	.	1
3	9	9	3
4	10	5	3
5	.	10	2
6	.	3	2

Merge variable

1 only in master file

2 only in user file

3 in both files

Merge: identifier

Master file

idpers	p07c44
1	8
2	7
3	9
4	10

using file

idpers	p08c44
3	9
4	5
5	10
6	3

idpers	p07c44	p08c44	_merge
1	8	.	1
2	7	.	1
3	9	9	3
4	10	5	3
5	.	10	2
6	.	3	2

Merge files: identifiers

	filename	identifiers
Individual master file	shp_mp	idpers, idhous$$, idfath__, idmoth__
Individual annual files	shp$$_p_user	idpers, idint, idhous$$, idspou__, refper$$
Additional ind. files (Social origin, last job, calendar, biographic)	shp_so, shp_lj shp_ca, shp0_*	idpers
Interviewer data	shp$$_v_user	idint
Household annual files	shp$$_h_user	idhous$$, refpers, idint, canton$$, (gdenr)
Biographic files		idpers
CNEF files	shpequiv_$$$$	x11101ll (=idpers)

The merge command

- Stata merge command

merge [type] [varlist] **using** filename [filename …] [, options]

varlist	identifier(s), e.g. idpers
filename	data set to be merged

type

1:1	each observation has a unique identifier in both data sets
1:m, m:1	in one data set several observations have the same identifier

1:1 merge individual files

2 annual individual files

```
use shp08_p_user, clear
merge 1:1  idpers using shp00_p_user
```

```
 _merge |      Freq.     Percent        Cum.
------------+-----------------------------------
          1 |      5,845       34.93       34.93
          2 |      5,056       30.21       65.14
          3 |      5,833       34.86      100.00
------------+-----------------------------------
      Total |     16,734      100.00
```

1:1 merge master file

annual **individual** file and individual **master** file

```
use shp08_p_user, clear // opens the file (master)
count  //there are 10’889 cases
merge 1:1 idpers using shp_mp //identif. & using file
tab _merge


     _merge |      Freq.     Percent        Cum.
------------+-----------------------------------
          2 |     11,111       50.50       50.50
          3 |     10,889       49.50      100.00
------------+-----------------------------------
      Total |     22,000      100.00

drop if _merge==2 //if only ind. from 2008 wanted
drop _merge
```

m:1 merge

annual individual file and annual household file

```
use shp08_p_user, clear //master file
merge m:1 idhous08 using shp08_h_user
/*identifier & using file */
```

```
     _merge |      Freq.     Percent        Cum.
------------+-----------------------------------
          3 |     10,889      100.00      100.00
------------+-----------------------------------
      Total |     10,889      100.00
```

More on merge

- Options of merge command
  - **keepusing** (varlist): selection of variables from using file
  - **keep**: selection of observations from master and/or using file
  - for more options: type **help merge**

- Merge many files
  - loops (see handout)

- Create partner files (see handout)

Wide and long format

- Wide format

idpers	i04empyn	i05empyn	i06empyn	i07empyn
4101	103190	107730	113400	122470
42101	63180	69500	.	.
56102	35473	.	41400	45500

- Long format (person-period-file)

idpers	year	iempyn
4101	2004	103190
4101	2005	107730
4101	2006	113400
4101	2007	122470
42101	2004	63180
42101	2005	69500
56102	2004	35473
56102	2006	41400
56102	2007	45500

Use of long data format

- All panel applications: xt commands
  - descriptives
  - panel data models
    - fixed effects models, random effects, multilevel
    - discrete time event-history analysis

- declare panel structure
  panel identifier, time identifier
  xtset idpers wave

Convert wide form to long form

`reshape long` command in stata

reshape long *varlist*, i(idpers) j(wave)

But: stata does not automatically detect years in varname

```
reshape long i@wyn p@w32 age@ status@, ///
 i(idpers) ///
 j(wave "99" "00" "01" "02" "03" "04" ///
 "05" "06" "07" "08" "09" "10"), atwl()
```

Create a long file with append

1. Modify dataset for each wave

2. Stack data sets:

```
use temp1, clear
forval y = 2/10 {
append using temp`y'
}
```

idpers	i99wyn
1	50’000
2	24’800
3	108’000

temp1.dta

idpers	wave	iwyn
1	1	50’000
2	1	24’800
3	1	108’000

idpers	i00wyn
1	51’000
2	25’800
3	109’000

temp2.dta

idpers	wave	iwyn
1	2	51’000
2	2	25’800
3	2	109’000

⋮

idpers	i10wyn
1	52’000
2	26’800
7	11’000

temp10.dta

idpers	wave	iwyn
1	10	58’000
2	10	26’800
7	10	11’000

Work with time lags

- If data in long format and defined as panel data (xtset)
- l. indicates time lag
- f. indicates time lead

- Example:
  social class of last job (see handout)
  life events

Missing data in the SHP

Missing data in the SHP: negative values

- -1 does not know
- -2 no answer
- -3 inapplicable (question has not been asked)
- -8/-4 other missings

Missing data in Stata: . .a .b .c .d etc.

- negative values are treated as real values
- missing data (. .a .b etc.) are defined as the highest possible values; . < .a < .b < .c < .d

→ recode to missing or analyses only positive values

e.g. sum i08empyn if i08empyn>=0

→ care with operator >

e.g. count if i08empyn>100000 counts also missing values

→ write <. instead of !=.

Longitudinal data analysis with Stata

xt commands:

descriptive statistics

- xtdescribe
- xtsum, xttab, xttrans

regression analysis

- Random Intercept: xtreg, xtgls, xtlogit, xtpoisson, xtcloglog
- Random Slope: mixed, melogit, ...

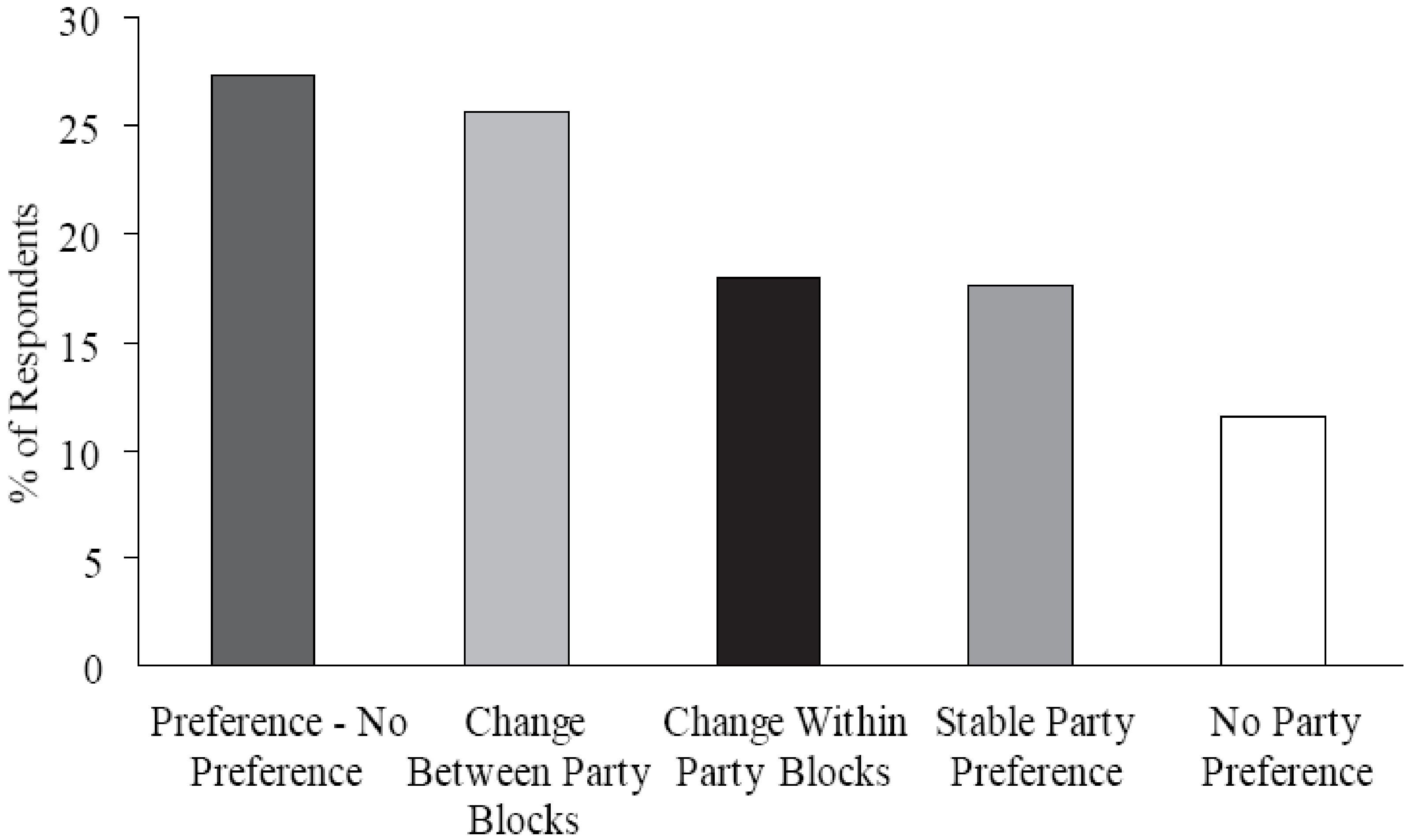
diagrams: xtline

Descriptive analysis

- Get to know the data
- Usually: similar findings to complicated models
- Visualisation
- Accessible results to a wider public
- Assumptions more explicit than in complicated models

Example: variability of party preferences

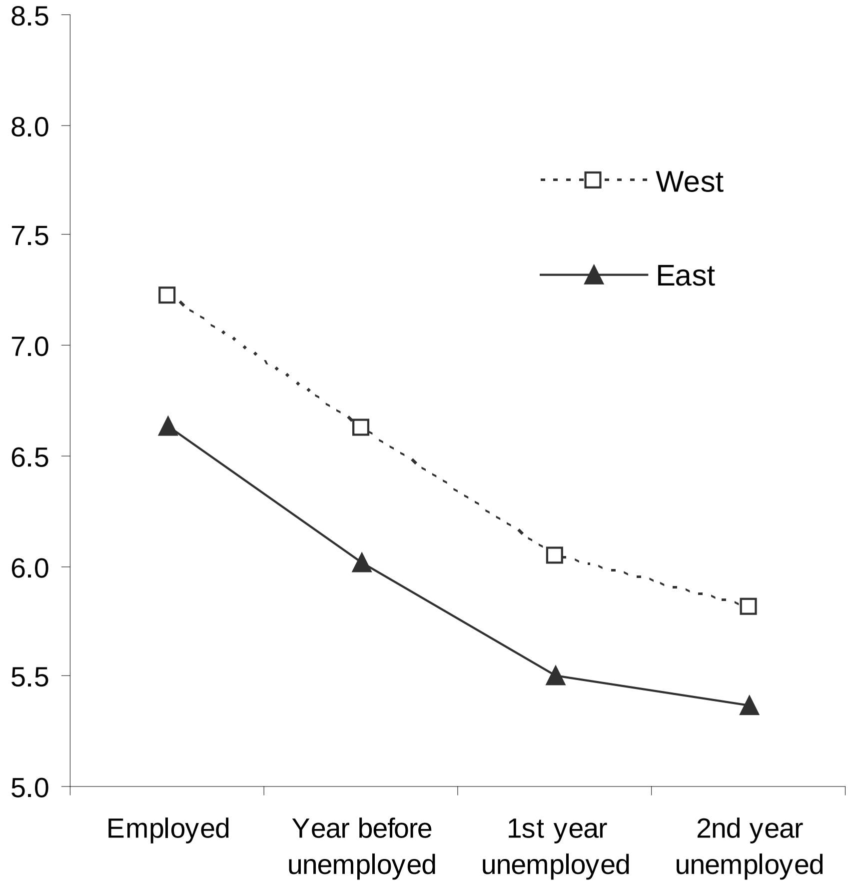
Figure 1: Patterns of Change over Nine Yearly Observations: Frequencies (N = 1'994)



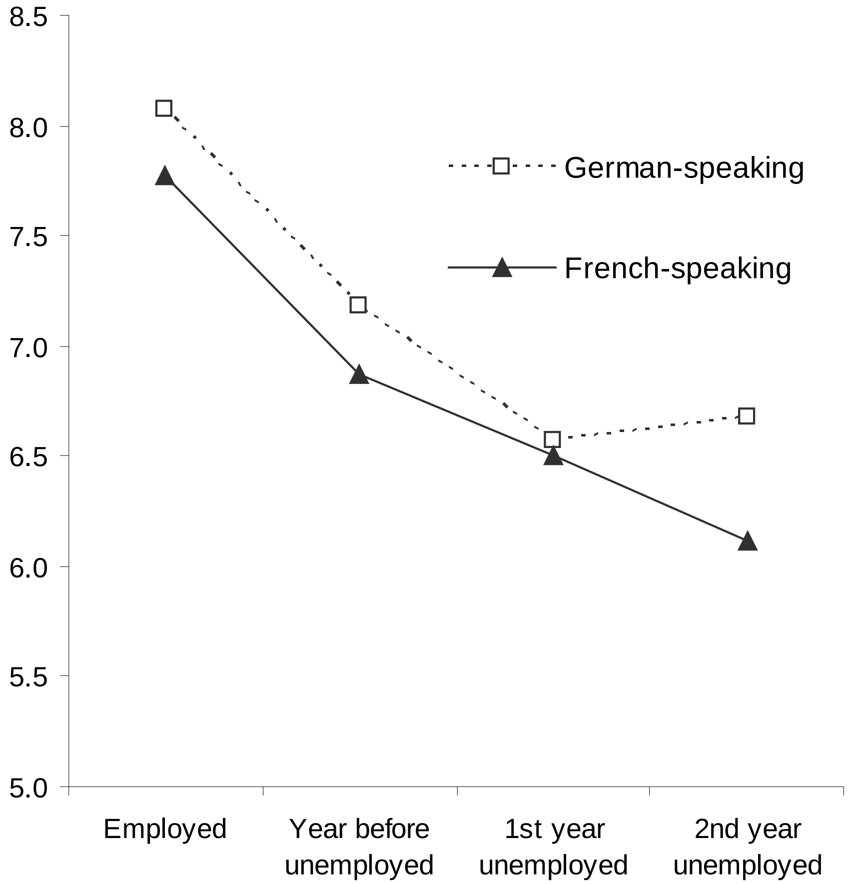
Kuhn (2009), Swiss Political Science Review 15(3): 463-494

Example: becoming unemployed

Satisfaction with life



Germany, 1984-2010



Switzerland, 2000-2010

Oesch and Lipps (2013), European Sociological Review 29(5): 955-967

Example: Income mobility

Switzerland	Low income 2009	Middle income 2009	High income 2009	Total
Low income 2005	56.2 %	40.8 %	3.1 %	100 %
Middle income 2005	13.5 %	75.8 %	10.8 %	100 %
High income 2005	4.4 %	34.4 %	61.1 %	100 %

Germany	Low income 2009	Middle income 2009	High income 2009	Total
Low income 2005	61.7 %	36.4 %	1.9 %	100 %
Middle income 2005	12.4 %	78.4 %	9.2 %	100 %
High income 2005	2.6 %	29.6 %	67.8 %	100 %

Grabka and Kuhn (2012), Swiss Journal of Sociology 38(2): 311–334

4
Linear regression
(Refresher course)

Aim and content

Refresher course on linear regression

- What is a **regression**?
- How to **obtain** regression coefficients?
- How to **interpret** regression coefficients?
- **Inference** from sample to population of interest (significance tests)

- Assumptions of linear regression
- Consequences when assumptions are violated

What is a regression?

A statistical method for studying the relationship between a single dependent variable and one or more independent variables.

Y: dependent variable

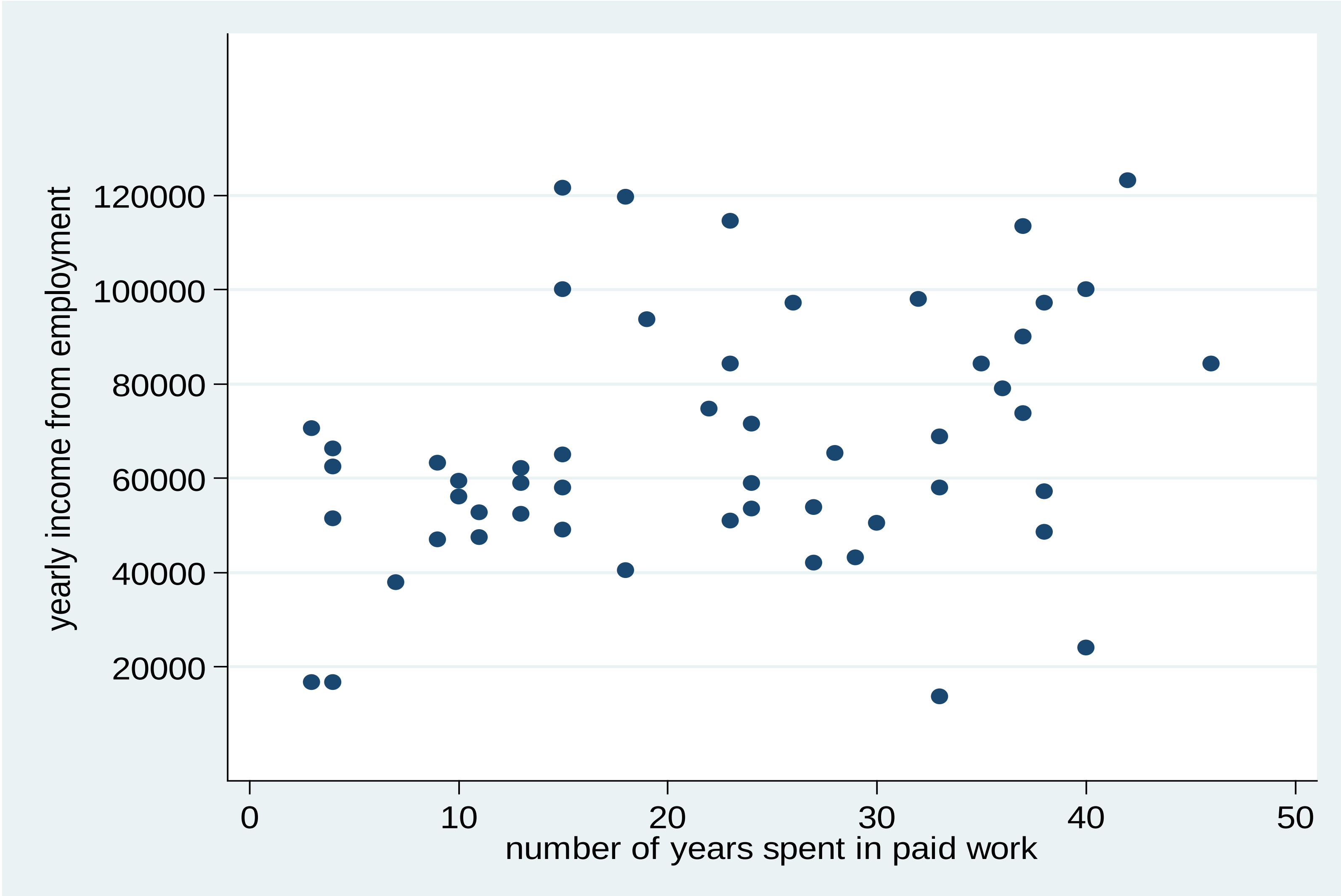
X: independent variable(s)

Simplest form: bivariate linear regression

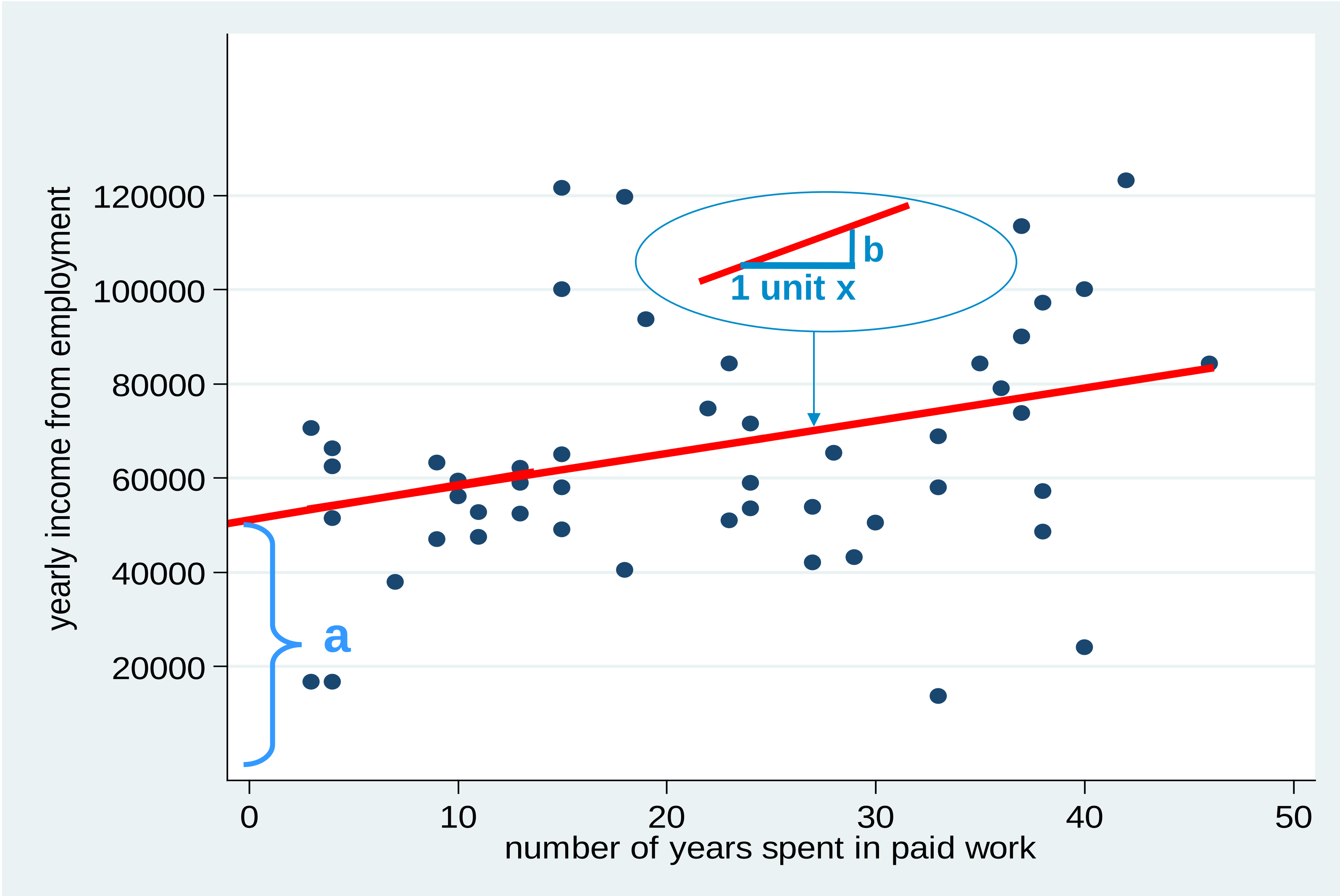
linear relationship between a dependent and one independent variable for a given set of observations

Examples

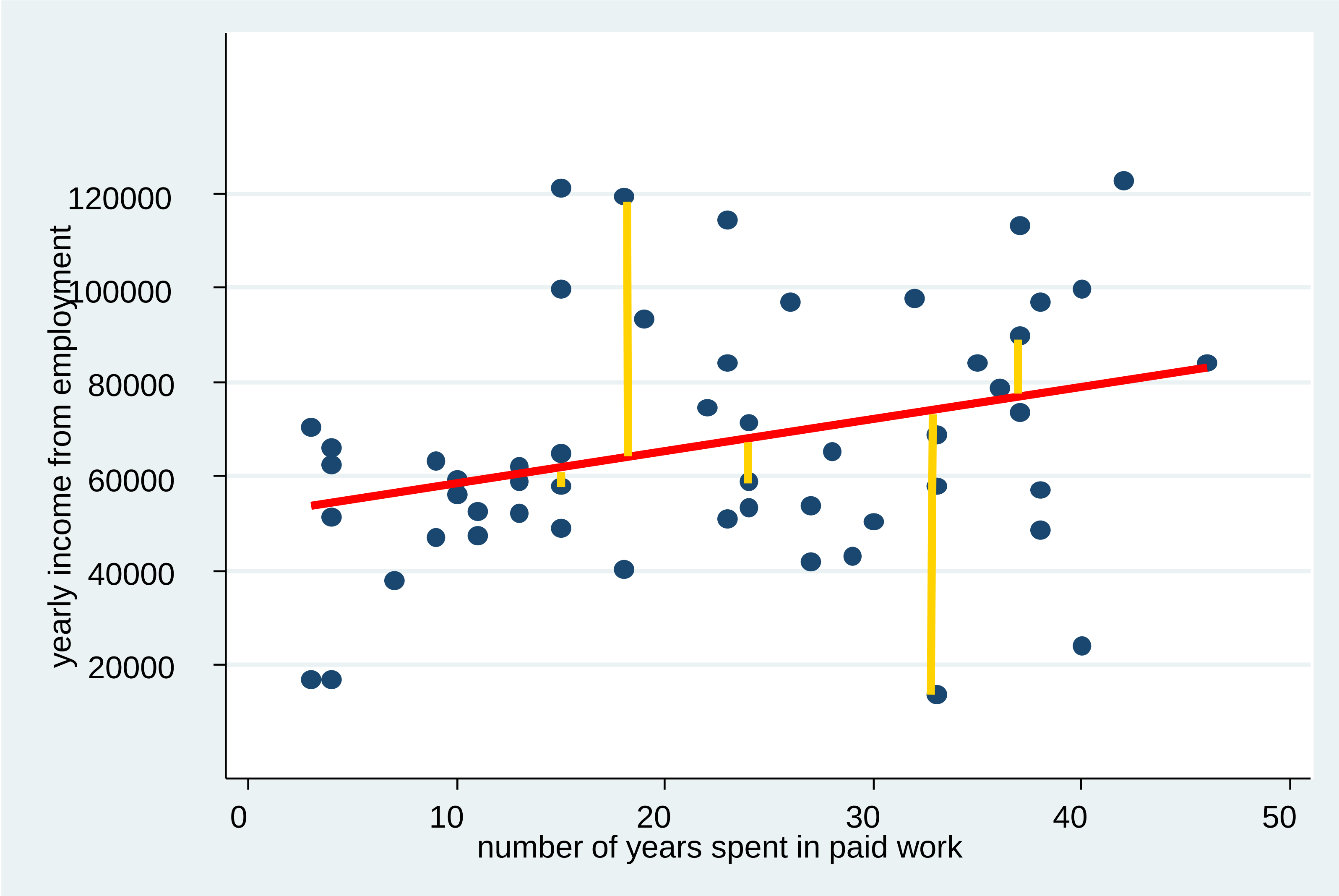
- Does the wage level affect the number of hours worked?
- Gender discrimination in wages?
- Do children increase happiness?



We start with a “scatter plot” of observations



Regression line: $\hat{y}_i = a + bx_i =$ **$51375 + 693 * x_i$**



Regression line: $\hat{y}_i = a + b^*x_i =$ **51375 + 693*xi**

Estimated regression equation: $y_i = \quad a \quad + b \quad x_i + \mathbf{e_i}$

Components of (linear) regression equation

Estimated regression equation:

univariate: $y = a + bx + e$

multivariate: $y = a + b_1x_1 + b_2x_2 + b_3x_3 + \ldots + e$

y dependent variable

x independent variable(s) (predictor(s), regressor(s))

a intercept (predicted value of Y if x=0)

b regression coefficients (slope): measure of the effect of X on Y

multivariate regression: the portion of y explained by x that is not explained by the others x's

e part of y not explained by x (residual), due to

omitted variables, measurement errors, stochastic shock, disturbance

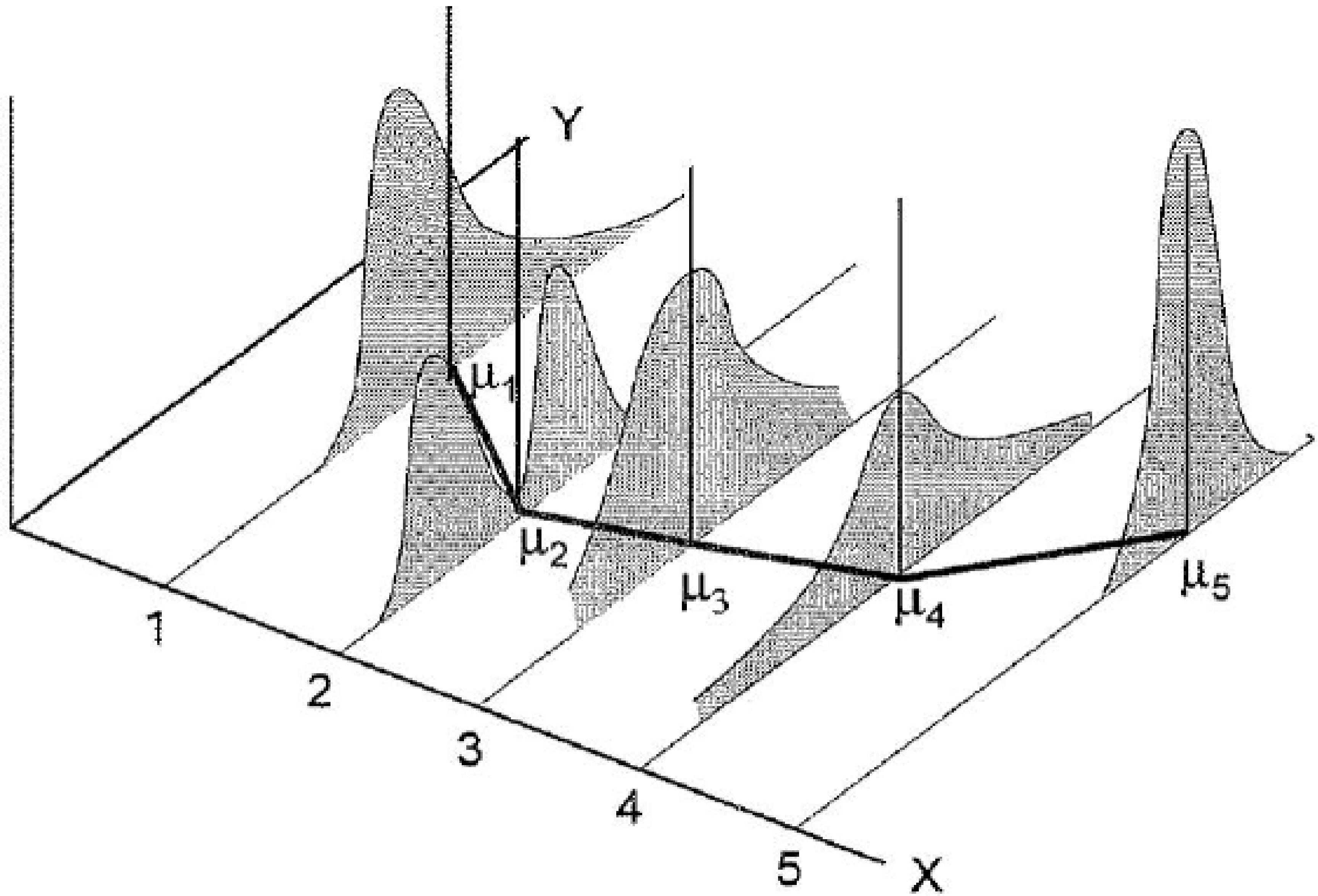
We assume a **linear** relationship between the conditional expectation value of Y and X

Scales of independent variables

- Continuous variables: linear
- Binary variables (Dummy variables) (0, 1)
  Example: female=1, male=0
- Ordinal or multivariate variables (n categories)
  Create n-1 dummy variables (base category)

  Example: educational levels
  - 1 low educational level
    2 intermediate educational level
    3 high educational level
  - Include 2 dummy variables in regression model

Regression – graphical interpretation



Example: gender wage gap

- Sample: full-time employed, yearly salary between 20’000 and 200’000 CHF

	Bivariate	Multivariate
Constant	98790	45'369
Female	-17'737	-9'090
Education (Ref.: compulsory)		
Secondary		9'197
Tertiary		30'786
Supervision		17'128
Financial sector		15'592
Number of years in paid work		729

Test if $\beta \neq 0$

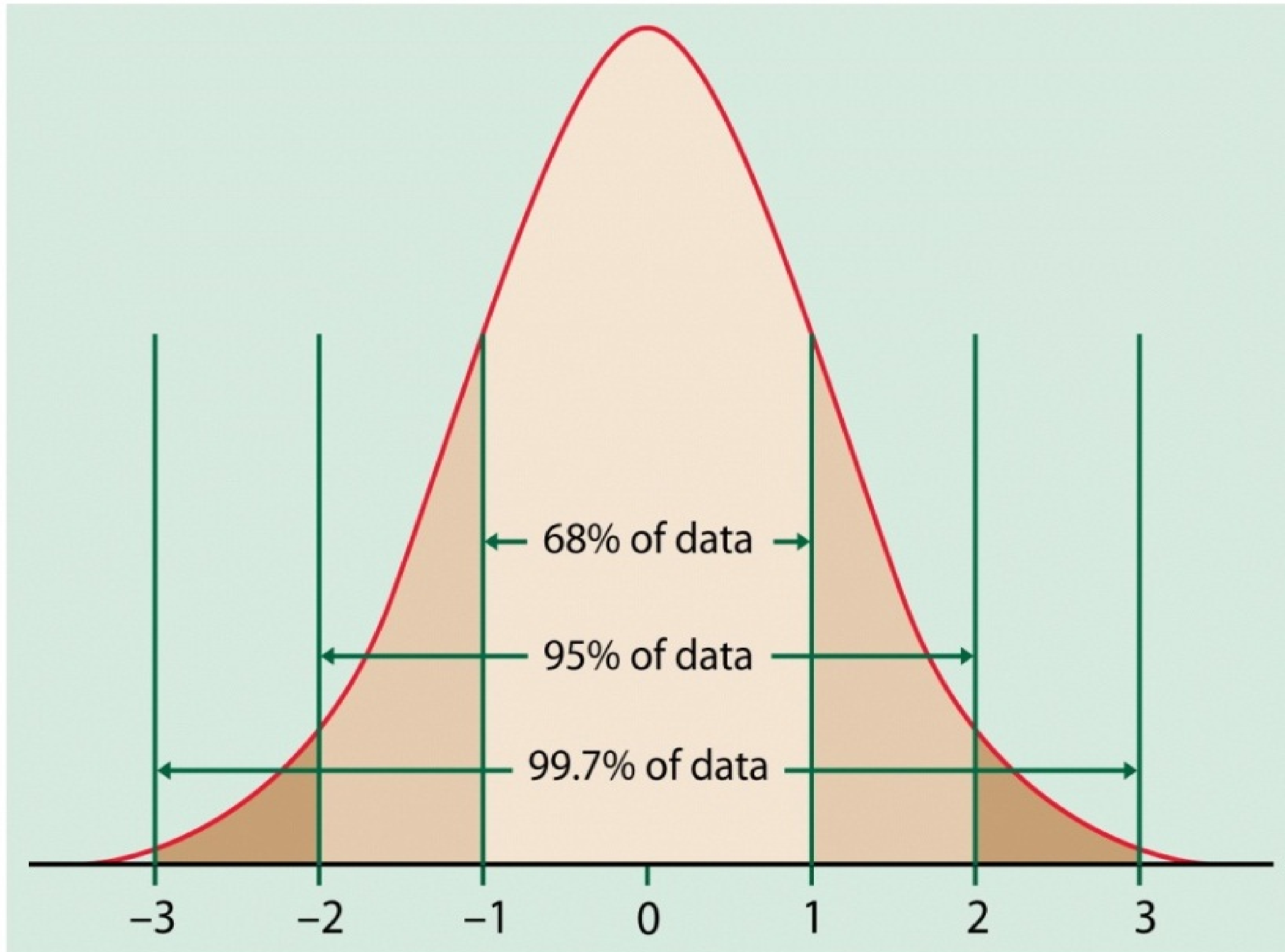
If β=0 (in population), there is **no relationship between x and y**

→ $\mathbf{H_0: \beta = 0}$

- H_0: Distribution if β=0

→ compare estimate with critical value

→ if abs(b) > abs(critical value): b significant



Test: **β / s.e.(β)** is t-distributed.

Rule of thumb: if > 2, then significant on 5% level. σ_b

Regression with Stata: cross-sectional regression

Example: life satisfaction, SHP data 2012

```
. reg lifesat partner age agesq edulow eduhigh lnincome

      Source |       SS       df       MS              Number of obs =    6916
-------------+------------------------------           F(  6,  6909) =   58.64
       Model |  640.854082     6  106.809014           Prob > F      =  0.0000
    Residual |  12584.2476  6909  1.82142822           R-squared     =  0.0485
-------------+------------------------------           Adj R-squared =  0.0476
       Total |  13225.1016  6915  1.91252374           Root MSE      =  1.3496

------------------------------------------------------------------------------
     lifesat |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     partner |   .5842025   .0404291    14.45   0.000     .5049491    .6634559
         age |  -.0617406   .0052466   -11.77   0.000    -.0720254   -.0514557
       agesq |   .0631447   .0052222    12.09   0.000     .0529076    .0733817
      edulow |  -.0318721    .046435    -0.69   0.492     -.122899    .0591549
     eduhigh |   .0765272   .0460582     1.66   0.097    -.0137611    .1668156
    lnincome |   .2845323   .0343396     8.29   0.000     .2172162    .3518485
       _cons |   5.697941   .3903272    14.60   0.000      4.93278    6.463103
------------------------------------------------------------------------------
```

Inference: Variation of regression coefficient b

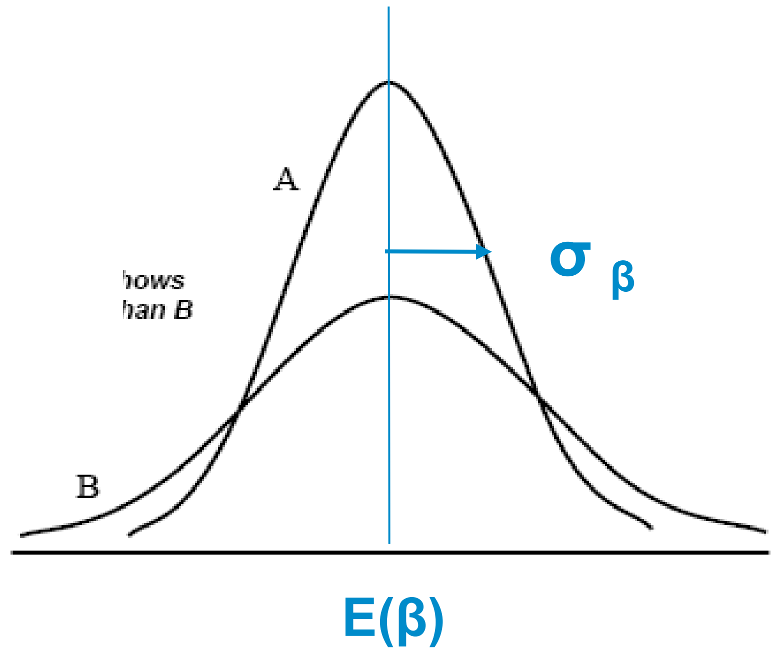
$$V(b) = \sigma_\beta^2 = \frac{\sigma_\varepsilon^2}{\sum (x_i - \bar{x})^2} \qquad \sigma_\varepsilon^2 = \frac{\sum (\varepsilon_i)^2}{n-p}$$

Variation of b (σ_β^2): decreases if

- n increases
- x are more spread out
- squared residuals decrease

Distribution of b

- Student t-distribution
  = normal distribution if n large



Example: significance levels and sample size

Sample n=53

	Coef.	st.e.	t	P>|t|	[95% Conf.	Interval]
years work.	692.6	289.1	2.40	0.020	112.1	1273.0
_cons	51375.9	7340.4	7.00	0.000	36639.4	66112.5

R^2: 0.101

Sample n=1787

	Coef.	St.e.	t	P>|t|	[95% Conf.	Interval]
years work.	931.4	50.6	18.40	0.000	832.1	1030.7
_cons	51218.6	1271.2	40.29	0.000	48725.5	53711.7

R^2: 0.159

Note: Standard error has same scale as coefficient

Assumptions of OLS regression

General

Continuous dependent variable

Random sample

Coefficient estimation

No perfect multicollinearity

E(e) = 0 (artifact)

No endogeneity; Cov(x,e) = 0

Coefficients biased (inconsistent)

Inference

- No autocorrelation $Cov(e_i,e_k)=0$
- Constant variance (no heteroscedasticity)
- Preferentially: residuals normally distributed

Standard errors of coefficients biased

Inference : assumptions

Assumptions on error terms

- Independence of error terms , no autocorrelation: Cov (ε_i, ε_k) = 0 for all i,k, i≠k
- Constant error variance : Var(ε_i)=σ^2_ε for all i; (Homoscedasticity)

Preferentially: e is normally distributed

Matrix of error terms

$i;k$	1	2	3	4	5	$\cdots$	$n-1$	n
1	σ^2	0	0	0	0	$\cdots$	0	0
2	0	σ^2	0	0	0	$\cdots$	0	0
3	0	0	σ^2	0	0	$\cdots$	0	0
4	0	0	0	σ^2	0	$\cdots$	0	0
5	0	0	0	0	σ^2	$\cdots$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	0	0
$n-1$	0	0	0	0	0	$\cdots$	σ^2	0
n	0	0	0	0	0	$\cdots$	0	σ^2

Autocorrelation

Reason: Nested observations (e.g. households, schools, time, municipalities)

$\rightarrow$ standard errors underestimated

What to do: OLS with adjust standard errors

autocorrelation

$i;k$	1	2	3	4	5	$\cdots$	$n-1$	n
1	$\boldsymbol{\sigma_1}^2$	$\boldsymbol{\sigma_2}^2$	$\boldsymbol{\sigma_2}^2$	0	0	$\cdots$	0	0
2	$\boldsymbol{\sigma_2}^2$	$\boldsymbol{\sigma_1}^2$	$\boldsymbol{\sigma_2}^2$	0	0	$\cdots$	0	0
3	$\boldsymbol{\sigma_2}^2$	$\boldsymbol{\sigma_2}^2$	$\boldsymbol{\sigma_1}^2$	0	0	$\cdots$	0	0
4	0	0	0	$\boldsymbol{\sigma_1}^2$	$\boldsymbol{\sigma_2}^2$	$\cdots$	0	0
5	0	0	0	$\boldsymbol{\sigma_2}^2$	$\boldsymbol{\sigma_1}^2$	$\cdots$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	0	0
$n-1$	0	0	0	0	0	$\cdots$	$\boldsymbol{\sigma_1}^2$	$\boldsymbol{\sigma_2}^2$
n	0	0	0	0	0	$\cdots$	$\boldsymbol{\sigma_2}^2$	$\boldsymbol{\sigma_1}^2$

no autocorrelation

$i;k$	1	2	3	4	5	$\cdots$	$n-1$	n
1	σ^2	0	0	0	0	$\cdots$	0	0
2	0	σ^2	0	0	0	$\cdots$	0	0
3	0	0	σ^2	0	0	$\cdots$	0	0
4	0	0	0	σ^2	0	$\cdots$	0	0
5	0	0	0	0	σ^2	$\cdots$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	0	0
$n-1$	0	0	0	0	0	$\cdots$	σ^2	0
n	0	0	0	0	0	$\cdots$	0	σ^2

Endogeneity

- Traditional meaning: Variable is determined within a model
- Here (econometric): Any situation where an explanatory variable is correlated with the residual Cov(x,e) = 0
- Reasons
  - Omitted variables
  - Measurement error
    in explanatory variables: underestimated effects,
    in dependent variable: larger variance of error term
  - Simultaneity
  - Nonlinearity in parameters (can be corrected)
- If a variable is endogenous: **model cannot be interpreted as causal (bias)**

Omitted variable bias

x is correlated with an unobserved (omitted) variable

if this omitted variable is correlated with y (conditional on x)
-> all x's are biased

$y = a + bx + e$	b is the causal effect of x on y
$\quad = a + bx + (cx + e')$	if x is correlated with an unobserved variable
$\quad = a + (b+c)\,x + e'$	we estimate b+c instead of b (causal effect)

Example:

Causal model civic engagement → trust

Omitted variable

values, personality, childhood

civic engagement → trust

Endogeneity: simultaneity, reverse causality

Causal model $X \longrightarrow Y$

having a partner | life satisfaction

Simultaneity $X \rightleftarrows Y$

having a partner | life satisfaction

Consequence: Estimator biased

Detection and correction of endogeneity

- Difficult: caution for causal interpretation!
- Detection
  - Theory, literature (variable selection and interpretation) !!!!
  - Robustness checks
- Correction: instrumental variables, panel data (time ordering, within-models), structural equation modelling, discontinuity design….
- ! Overcontrol is common in social research based on regressions. Do not control for intervening mechanisms (“collider” variables)

Regression with panel data: Data structure

- Cross sectional data

idpers	lifestat04
1	5
2	9
3	3

- Panel data

idpers	year	lifesat
1	2004	5
1	2005	6
1	2006	7
1	2007	8
2	2004	9
2	2005	9
3	2004	3
3	2006	8
3	2007	5

OLS with pooled panel data: problems I

- Pooled data: long data format, different years in one file
- Problem: OLS assumption of independent observations violated (autocorrelation)

  → coefficients unbiased

  → but standard errors biased (underestimation)
- Possible measure: Correct for clustering in error terms
- But: OLS is not the best estimator for pooled data (not efficient)

OLS with pooled panel data: example

Example: Partner -> Life satisfaction

- SHP 2000-2012
- 80’914 observations from 14’345 individuals

Life satisfaction	OLS		OLS (correct for cluster)	
Partner	0.481***	(.012)	0.481***	(.024)
Age	-0.070***	(.002)	-0.070***	(.003)
Age squared	0.077***	(.002)	0.077***	(.003)
Education: low	-0.016	(.013)	-0.016	(.027)
Education: high	0.009	(.013)	0.009	(.024)
Income (ln)	0.230***	(.009)	0.230***	(.015)
Health: so so	1.248***	(.037)	1.248***	(.077)
Health: well	2.025***	(0.035)	2.025***	(0.078)
Health: very well	2.502***	(0.036)	2.502***	(0.079)
Constant	4.564		4.564	

Stata: reg lifesat partner age agesq …. lnincome, cluster(idpers)

OLS with panel data: problems II and outlook

- OLS does not take advantage of panel structure
- Two different types of variation in panel data
  - Variation within individuals
  - Variation between individuals
- Control for unobservable variables (stable personal characteristics)
  - Within-models
  - Random Effect Models
    (multilevel /random intercept / hierarchical model/ frailty for event history, mixed model)

5
Causality

Interpretation of model results

- Descriptive interpretation

versus

- Causal interpretation

  Idea: use only variance in treatment variable which is exogenous (exogenously manipulated by researcher)

Causality

- **Def.:** Necessary (not sufficient) conditions for X to "cause" Y:
  - X **precedes** Y (also anticipation)
  - X **correlates** with Y
  - **theoretical** explanation of mechanism between X and Y ("law")

- Causality in social science **experiments**
  - Random group receives "treatment" (manipulation): no omitted variable bias (self-selection into treatment)
  - we have treatment and control group

Problems:

- Experiments usually not possible in social science (external validity): ethical or organizational problems
- What about effects of unchangeable variables (like sex)?
- Continuous variables?

How are causal effects analyzed?

- **Multiple Regression**: attempt to control for all omitted variables

  Problems: - omitted variables, unobserved heterogeneity

  - form of relationship must be specified

- **Propensity score matching**: attempt to compare members with same (or similar) scores on control variables

  Problem: - omitted variables, unobserved heterogeneity

  Advantage: - non-parametric

- **Instrumental variables:** use only variance of x that correlates with exogenous instrument z

- **Panel data**: before and after measurement

  Problems: - little before/after variation (within individuals)

  - co-varying change variables (corr. with ε_{it}) must be controlled

  Advantage: - co-varying change time-invariant variables (corr. with u_i) no longer a problem

Measuring causal effects (example: binary treat)

- **unbiased** Effects = $Y_{i,t_1}^{Treat} - Y_{i,t_1}^{NonTreat}$ counterfactual!

- Cross-sectional data: $Y_{i,t_1}^{Treat} - Y_{j,t_1}^{NonTreat}$ but: different persons i,j

- With **Panel data I: within-estimation** $Y_{i,t_1}^{Treat} - Y_{i,t_2}^{NonTreat}$

Causal effect but **problem with time-variant effects** (e.g., time, unmeasured within-changes with effects on Y)

Basic Approach

Between-estimation

- ok with experimental data
  - Due to randomization units differ only in the treatment
- But strong assumption of unit homogeneity causes bias
  - Problem: self-selection into treatment!
  - Unobserved unit heterogeneity

Within-estimation

- with control group often ok because the parallel trends assumption is much weaker
  - Unobserved unit heterogeneity will not bias within-estimation
  - Only differing time-trends in treatment and control group will bias within-estimation results

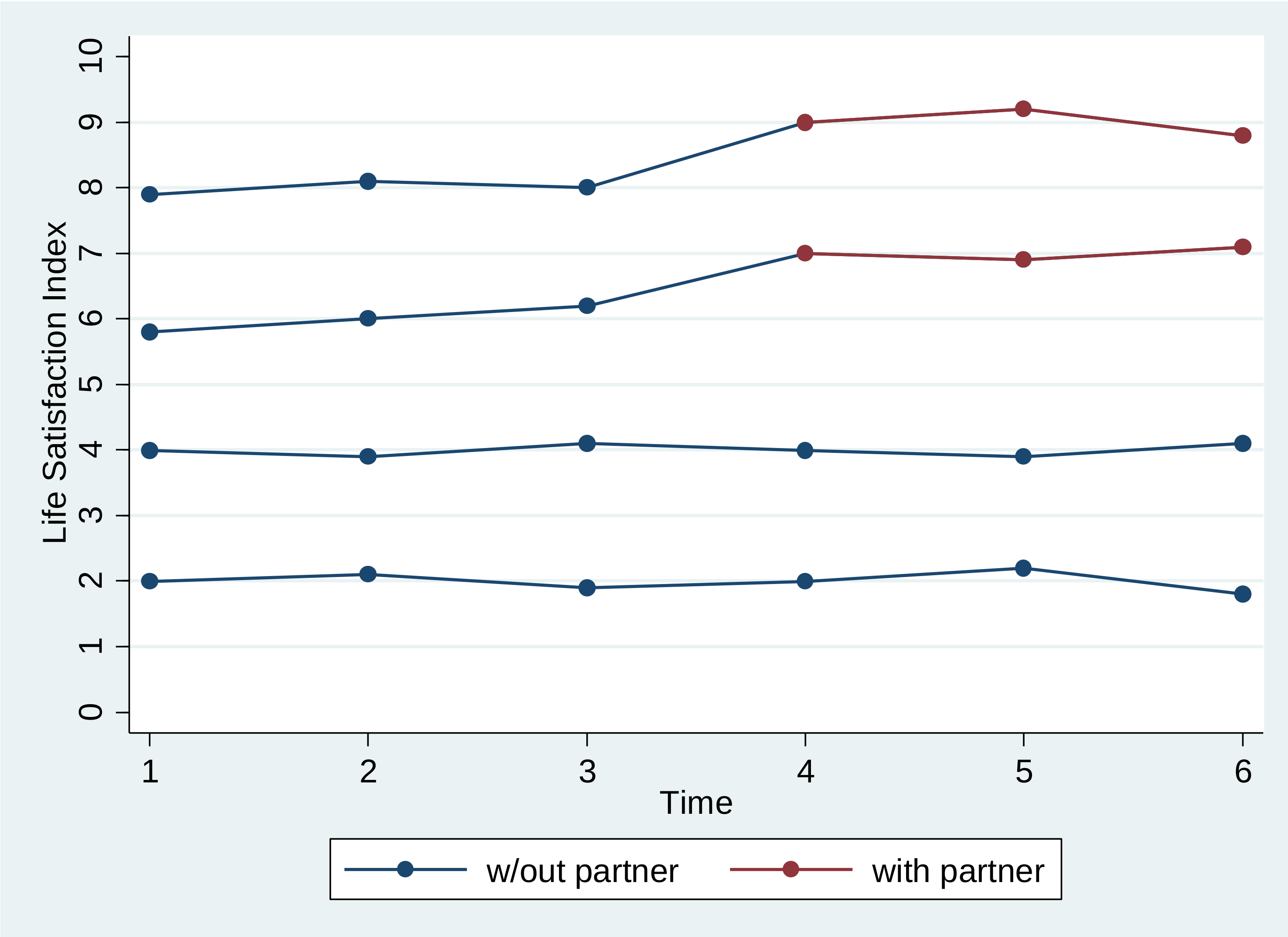
Hypothetical Data: does having a partner make happier?

```
. list id time satlife partner,
separator(6)
```

	id	time	satlife	partner
1.	1	1	2	0
2.	1	2	2.1	0
3.	1	3	1.9	0
4.	1	4	2	0
5.	1	5	2.2	0
6.	1	6	1.8	0
7.	2	1	4	0
8.	2	2	3.9	0
9.	2	3	4.1	0
10.	2	4	4	0
11.	2	5	3.9	0
12.	2	6	4.1	0

	id	time	satlife	partner
13.	3	1	5.8	0
14.	3	2	6	0
15.	3	3	6.2	0
16.	3	4	7	1
17.	3	5	6.9	1
18.	3	6	7.1	1
19.	4	1	7.9	0
20.	4	2	8.1	0
21.	4	3	8	0
22.	4	4	9	1
23.	4	5	9.2	1
24.	4	6	8.8	1

Problem: Self-Selection into Partnership



Causal effect

Selectivity: only the happier get a partner

Those with and those without partner differ in characteristics other than partnership: no unit homogeneity

Self-Selection: Treatment not under control

- Between-approach biases results
- Within-approach possible: we have before (t=1,2,3) and after (t=4,5,6) measurements
- Therefore unit heterogeneity no longer a problem
- We have in addition a control group: DiD

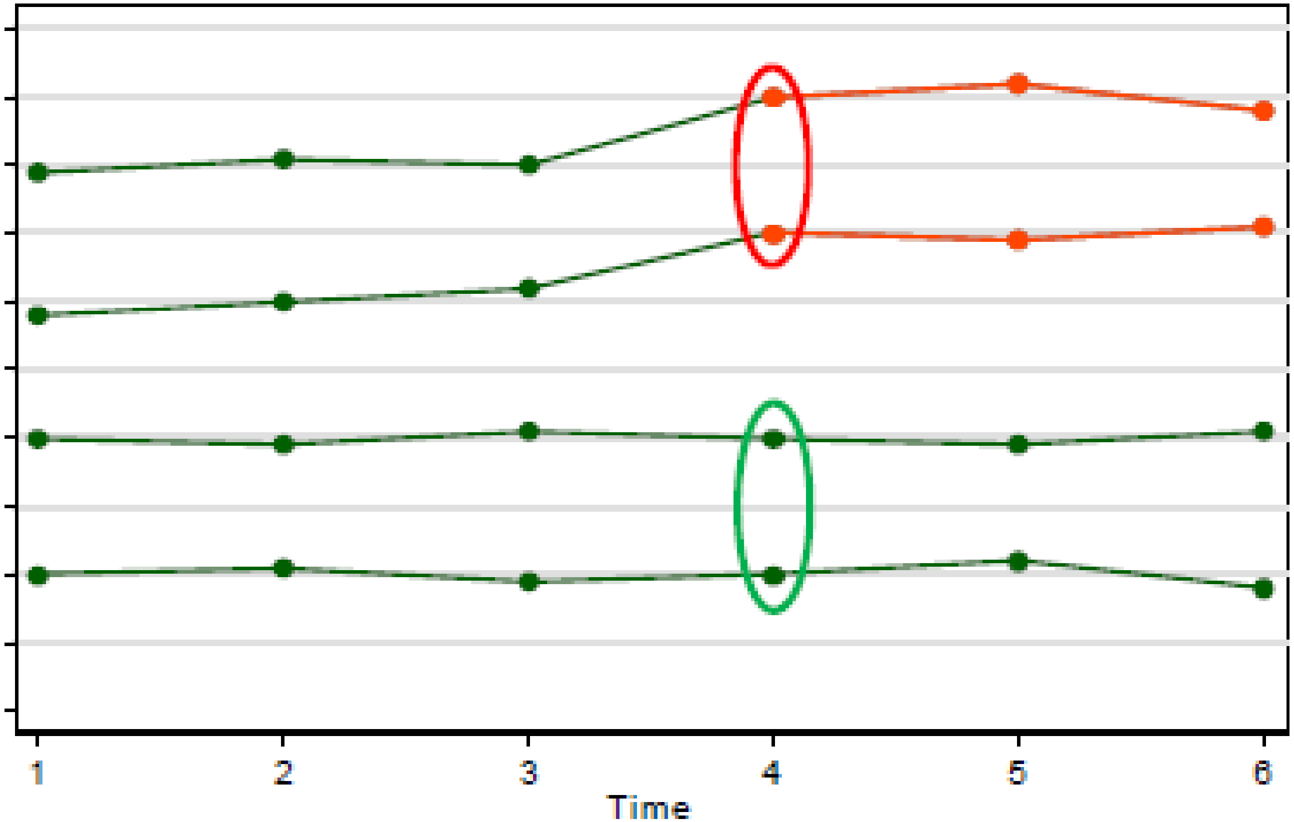
$$\text{after-before (treat)} = \text{sat}(t=4,5,6)-\text{sat}(t=1,2,3) \mid_{\text{treat}} = ((7-6)+(9-8))/2=1$$

$$\text{after-before (control)} = \text{sat}(t=4,5,6)-\text{sat}(t=1,2,3) \mid_{\text{control}} = ((2-2)+(4-4))/2=0$$

Average treatment effect: ATE: difference of averages of treatment and control group: ATE = 1

Can regression produce these results?

Cross-sectional regression at t=4



Cross-sectional regression at t=4:

mean circled red points – mean circled green points

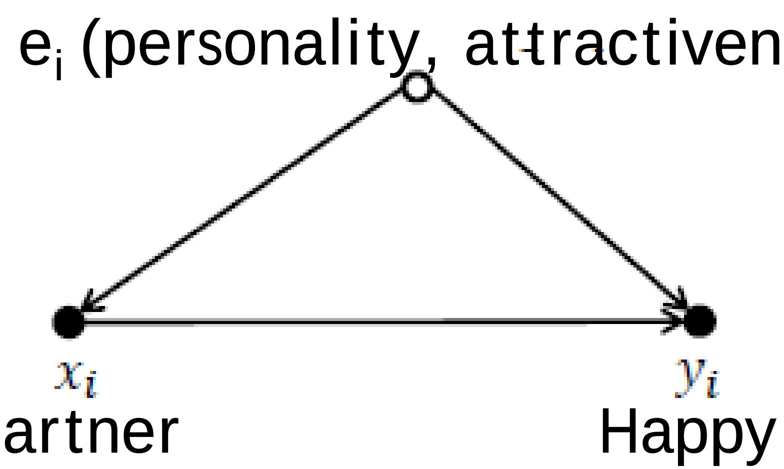
$\beta_{t=4}$ = (9+7)/2 – (4+2)/2 = 8–3 = 5

massively biased! Compares average happiness between partnered and unpartnered people

What is the problem?

most critical assumption of a linear regression is the **exogeneity assumption**: Cov(x,e) = 0

But: unobserved confounders (unobservables that affect both X and Y)

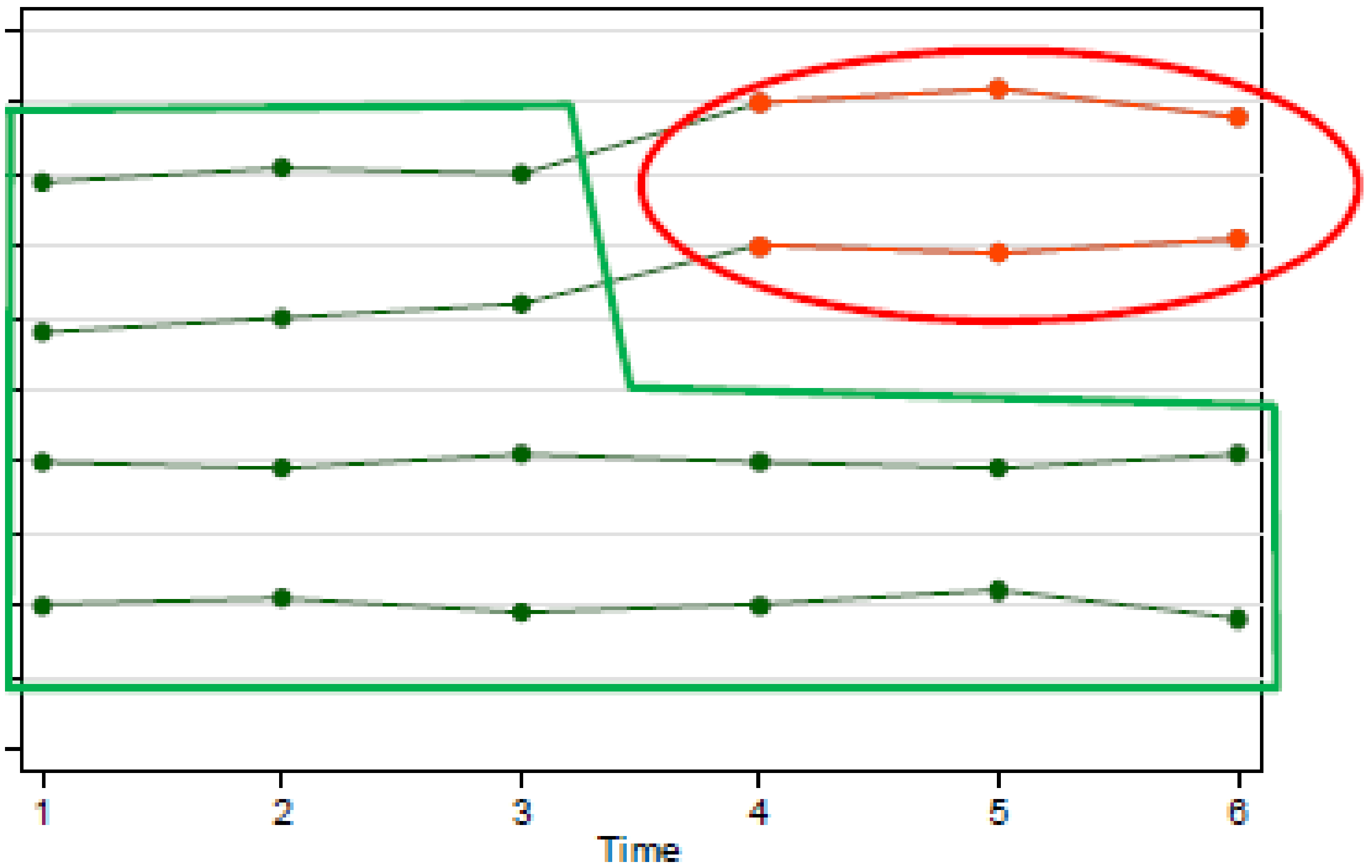


Cov(x,e) ≠ 0 (unobserved heterogeneity or **omitted variable bias**)

The happier self-select into partnership

Treatment and control group are not (initially) randomized

Pooled OLS is no solution



Pooled OLS:

mean red points – mean green points

Bias still high: $\beta_{pooled} = 3.67$

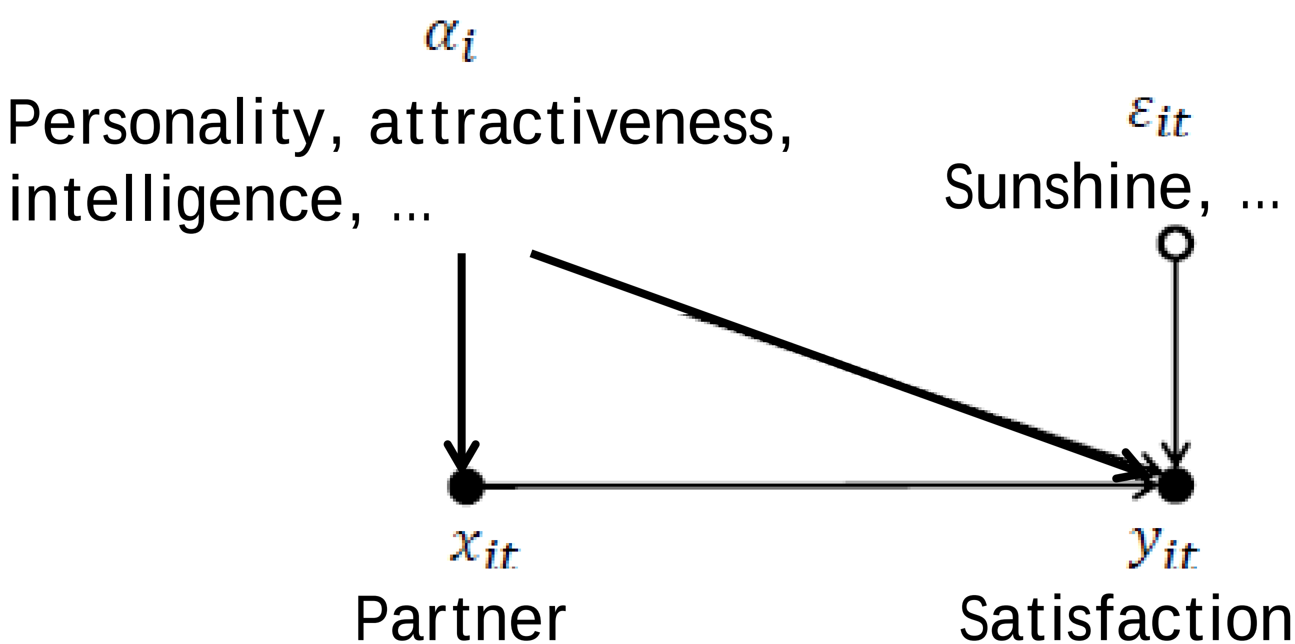
Pooled OLS still relies on between-comparison

Towards panel models: error decomposition

Starting point: error decomposition $e_{it} = \alpha_i + \varepsilon_{it}$

α_i person-specific time-constant error term («between»)
Assumption: person-specific random variable

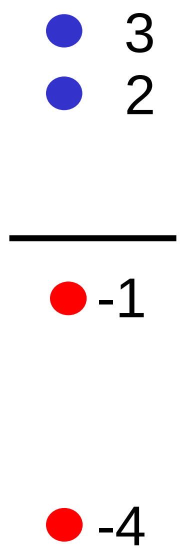
ε_{it} time-varying error term (idiosyncratic error term) ("within")
Assumption: zero mean, homoscedasticity, no autocorrelation



Excursus: total / within / between variance

$$\sum_{i=1}^{n}\sum_{t=1}^{T_i}(y_{it}-\overline{y})^2=\sum_{i=1}^{n}\sum_{t=1}^{T_i}(y_{it}-\overline{y}_i)^2+\sum_{i=1}^{n}\sum_{t=1}^{T_i}(\overline{y}_i-\overline{y})^2$$

$$\mathbf{T}_{yy} = W_{yy} + B_{yy}$$



Total (“Grand”) Mean (=0)

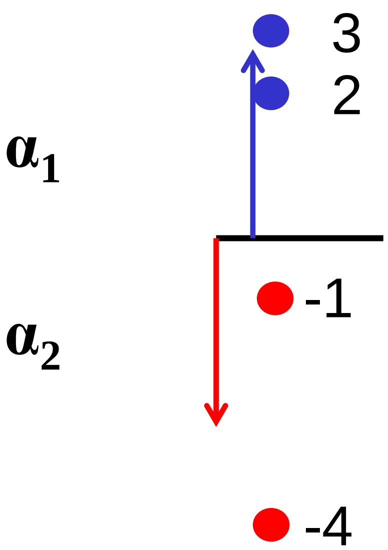
Total Variance is equal to the Square of the Differences of all Observations from the Total Mean divided by the Sample Size (4) =

$$\{ (3-0)^2 + (2-0)^2 + (-1-0)^2 + (-4-0)^2 \} / 4 = (9+4+1+16)/4 = 7.5$$

Illustration: between-variance

$$\sum_{i=1}^{n}\sum_{t=1}^{T_i}(y_{it}-\overline{y})^2 = \sum_{i=1}^{n}\sum_{t=1}^{T_i}(y_{it}-\overline{y}_i)^2 + \sum_{i=1}^{n}\sum_{t=1}^{T_i}(\overline{y}_i-\overline{y})^2$$

$$T_{yy} = W_{yy} + \mathbf{B}_{yy}$$



Total ("Grand") Mean (=0)

variance of the individual means = "**between**"-variance ($\sigma_{between} = \sigma_\alpha$)

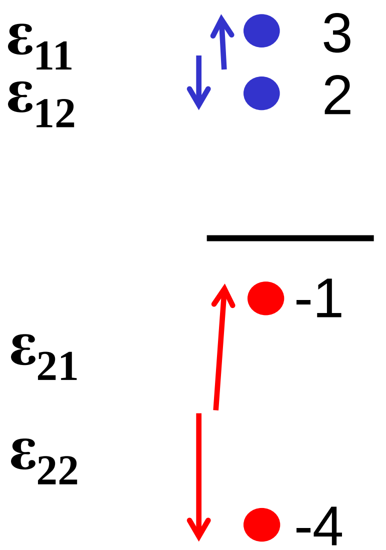
($\mathbf{2.5^2}$+$(-2.5)^2$)/2 = 6.25

(remember: total variance = 7.5)

Illustration: within-variance

$$\sum_{i=1}^{n}\sum_{t=1}^{T_i}(y_{it}-\bar{y})^2 = \sum_{i=1}^{n}\sum_{t=1}^{T_i}(\mathbf{y}_{it}-\bar{\mathbf{y}}_i)^2 + \sum_{i=1}^{n}\sum_{t=1}^{T_i}(\bar{y}_i-\bar{y})^2$$

$$T_{yy} = \mathbf{W}_{yy} + B_{yy}$$



Total (“Grand”) Mean (=0)

variance of measurements within individuals= “**within**”-Variance:

$((3-2.5)^2 + (2-2.5)^2 + (-1-(-2.5))^2 + (-4-(-2.5))^2\,)/4$

$=1.25$ (=18% of total variance)

variance of individual means = $(2.5^2+(-2.5)^2)/2 = 6.25$

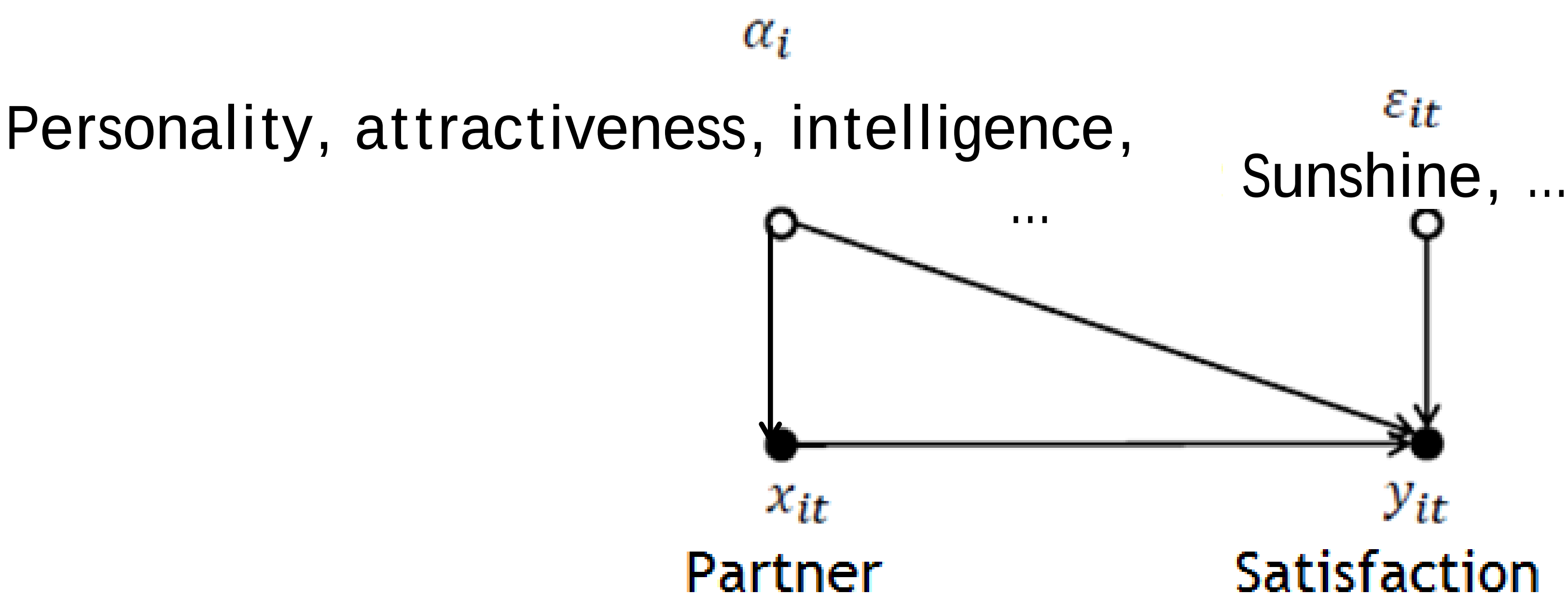
(=82% of total variance = ρ = **ICC** (intra-class-correlation))

Error components model

Model: $y_{it} = \beta x_{it} + \alpha_i + \varepsilon_{it}$

POLS is consistent only, if the regressor x_{it} is independent from **both** error components:

$E(\alpha_i \mid x_{it}) = 0$	no person-specific time-constant unobserved heterogeneity («random effects» assumption)
$E(\varepsilon_{it} \mid x_{it}) = 0$	no time-varying unobserved heterogeneity («strict exogeneity» assumption)



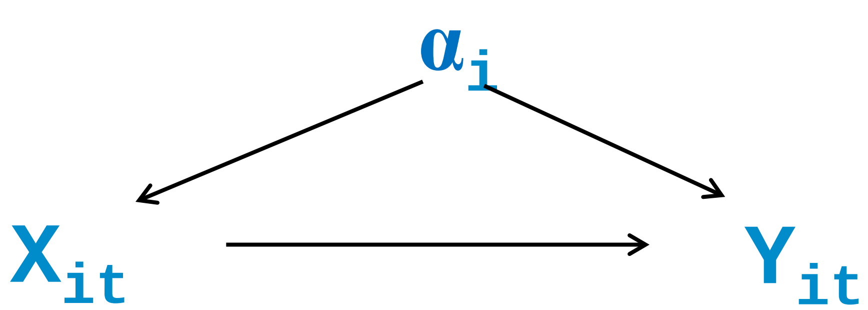
6

Fixed Effects (“within”) Models

Bias from omitted time-invariant variables

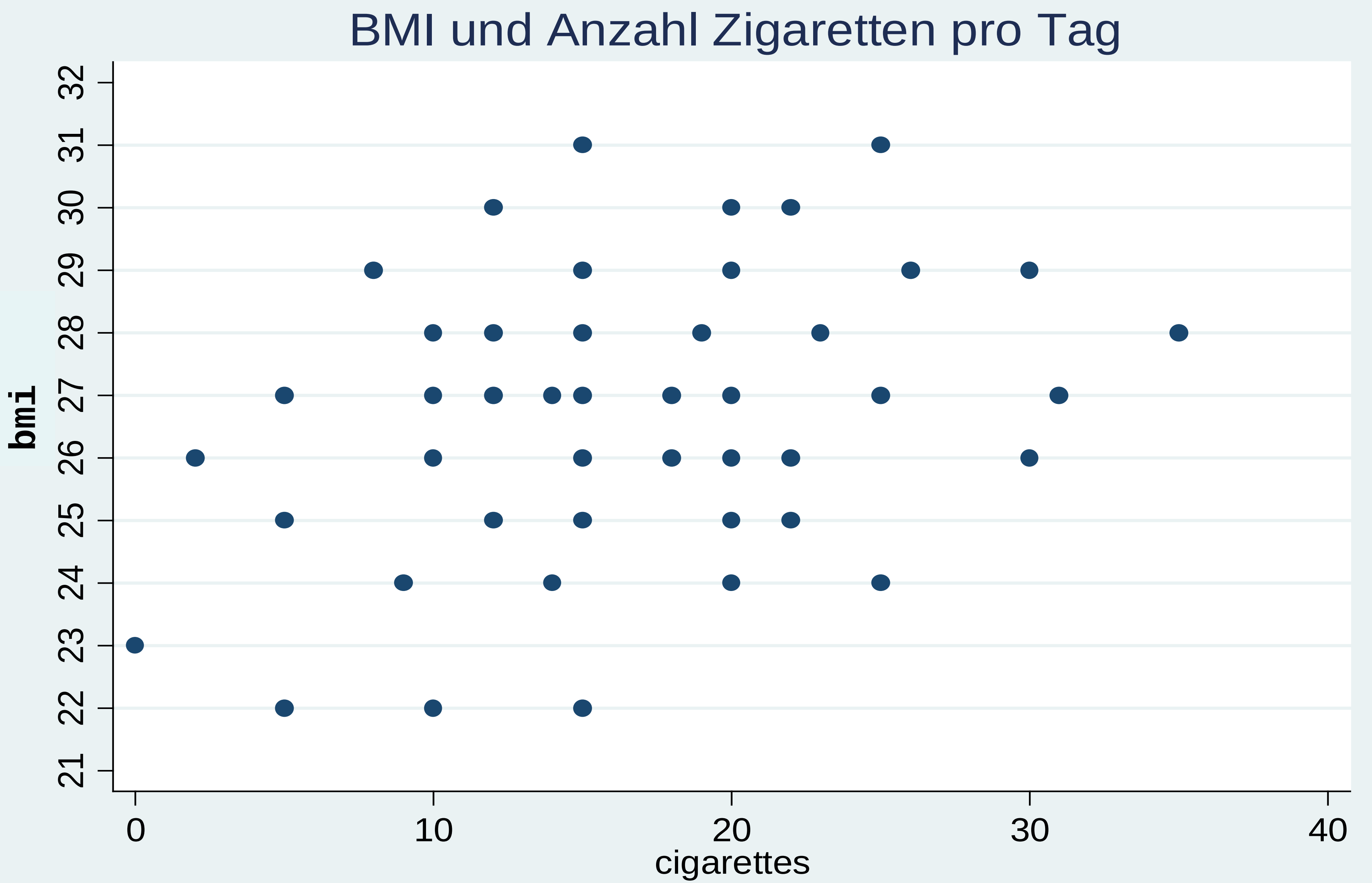
- Many *time-invariant* individual characteristics (α_i) are **not observed or not taken into account**
  - e.g. enthusiasm, ability, willingness to take risks, attractiveness
- These may have an effect on **dependent variable**, and are **correlated** with **independent variables** like satlife – partner - attractiveness)

  Then regression coefficients will be **biased!**

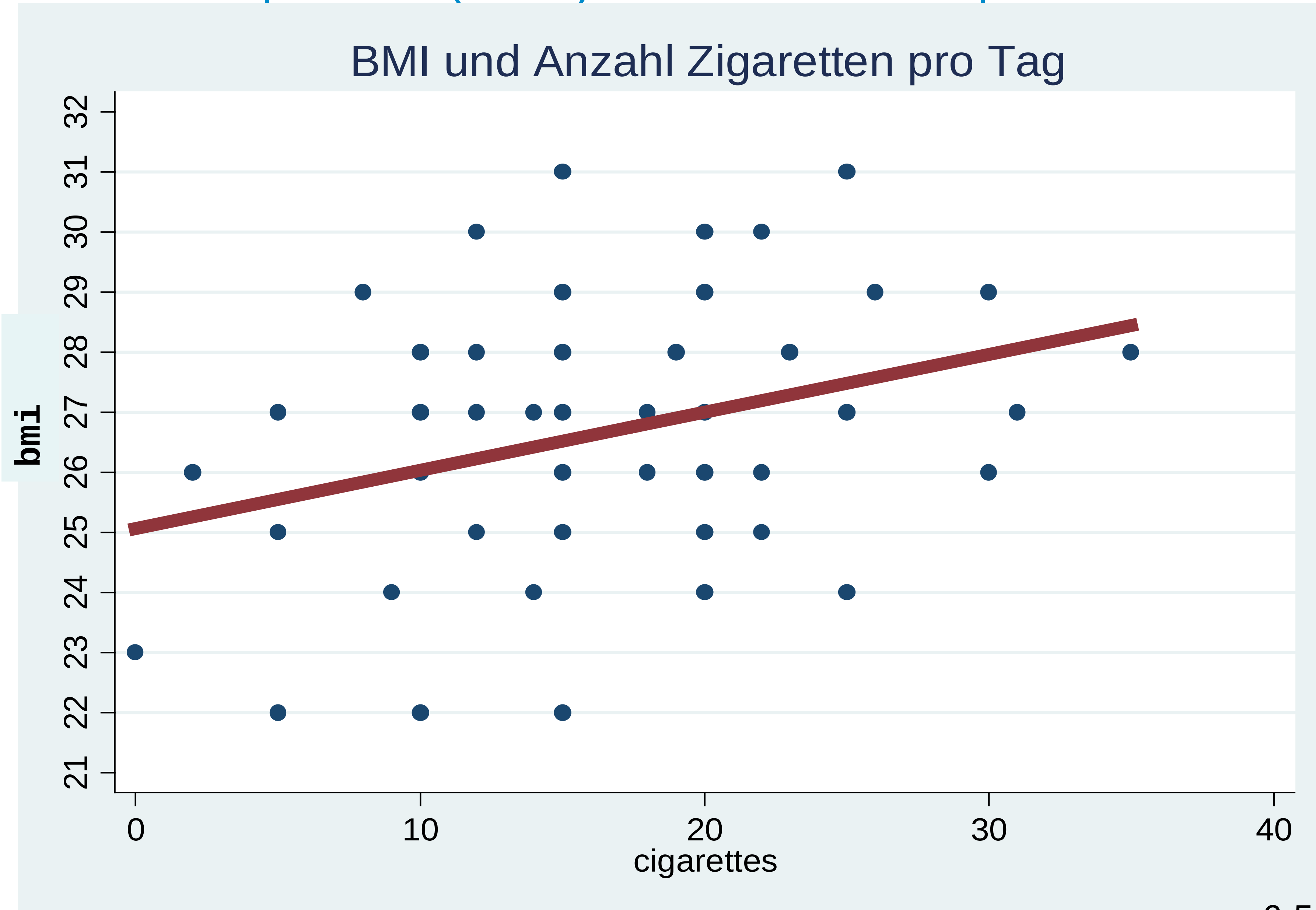


Hypothetical Example:
Omitted time-invariant Variable Bias
BMI (Y) and Smoking (X) :
Continuous "Treatment"

Hypothetical Observations: BMI and number of cigarettes per day



Scatter plot with (naive) linear fit line from pooled OLS



Pooled regression results

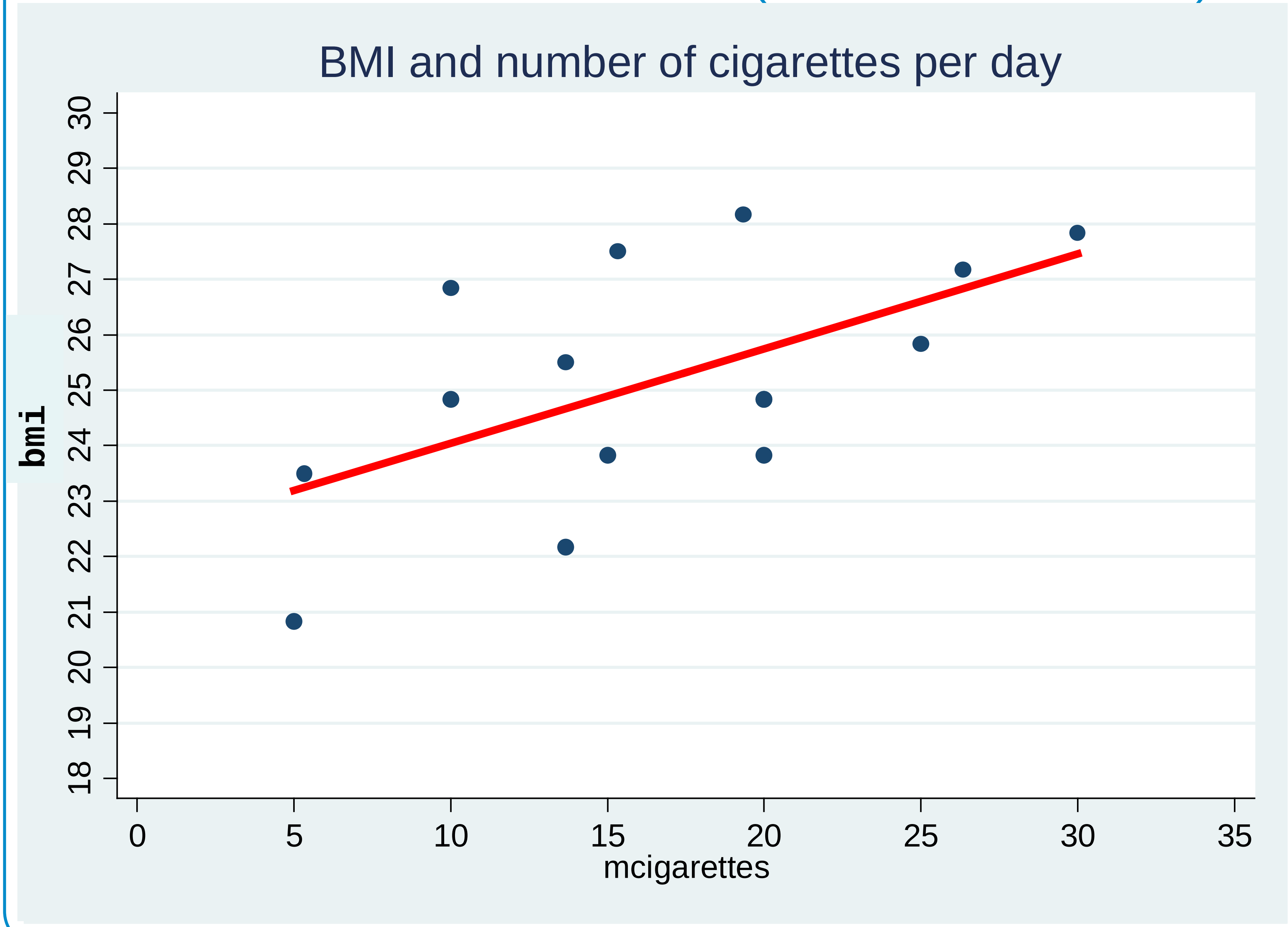
```
. * pooled regression:
. reg bmi cigarettes

      Source |       SS       df       MS              Number of obs =      45
-------------+------------------------------           F(  1,    43) =    5.26
       Model |  25.4934645     1  25.4934645           Prob > F      =  0.0268
    Residual |  208.506536    43   4.8489892           R-squared     =  0.1089
-------------+------------------------------           Adj R-squared =  0.0882
       Total |         234    44  5.31818182           Root MSE      =   2.202

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  cigarettes |   .0966882   .0421682     2.29   0.027      .011648    .1817285
       _cons |   25.06379   .7722906    32.45   0.000     23.50632    26.62126
```

BMI higher by .1 if number of cigarettes is 1 higher

Between-individual effects (means of individuals)



Between regression

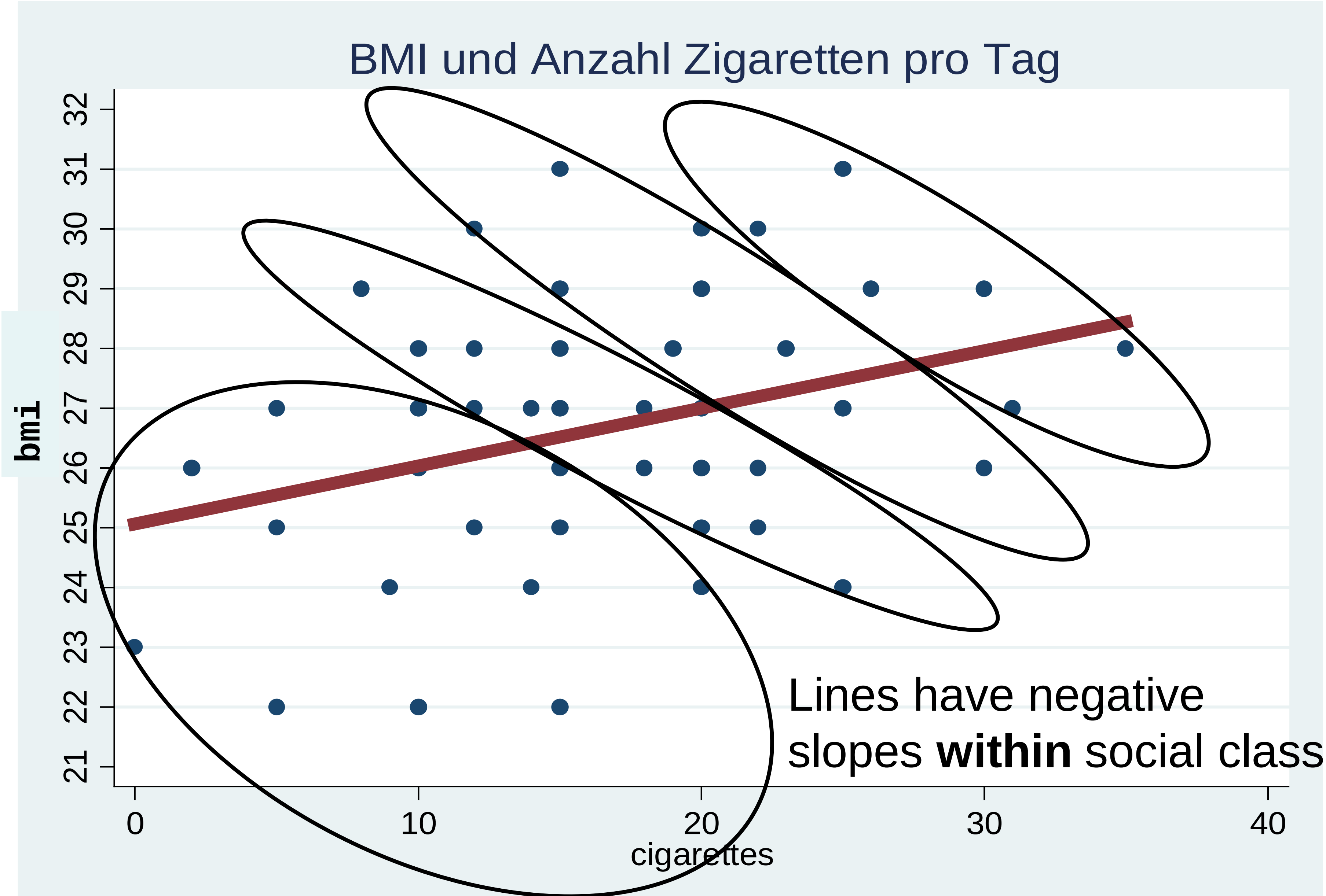
```
. * between-regression:
. egen mbmi=mean(bmi), by(id)
. egen mcigarettes=mean(cigarettes), by(id)
. bysort id: gen n=_n
. reg mbmi mcigarettes if n==1 // between-regression


      Source |       SS       df       MS              Number of obs =      15
-------------+------------------------------           F(  1,    13) =    6.98
       Model |   22.134779     1   22.134779           Prob > F      =  0.0203
    Residual |   41.198548    13  3.16911908           R-squared     =  0.3495
-------------+------------------------------           Adj R-squared =  0.2995
       Total |   63.333327    14  4.52380907           Root MSE      =  1.7802

------------------------------------------------------------------------------
        mbmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 mcigarettes |   .1709983   .0647029     2.64   0.020     .0312163    .3107803
       _cons |    23.8319   1.166966    20.42   0.000     21.31082    26.35297
------------------------------------------------------------------------------
```

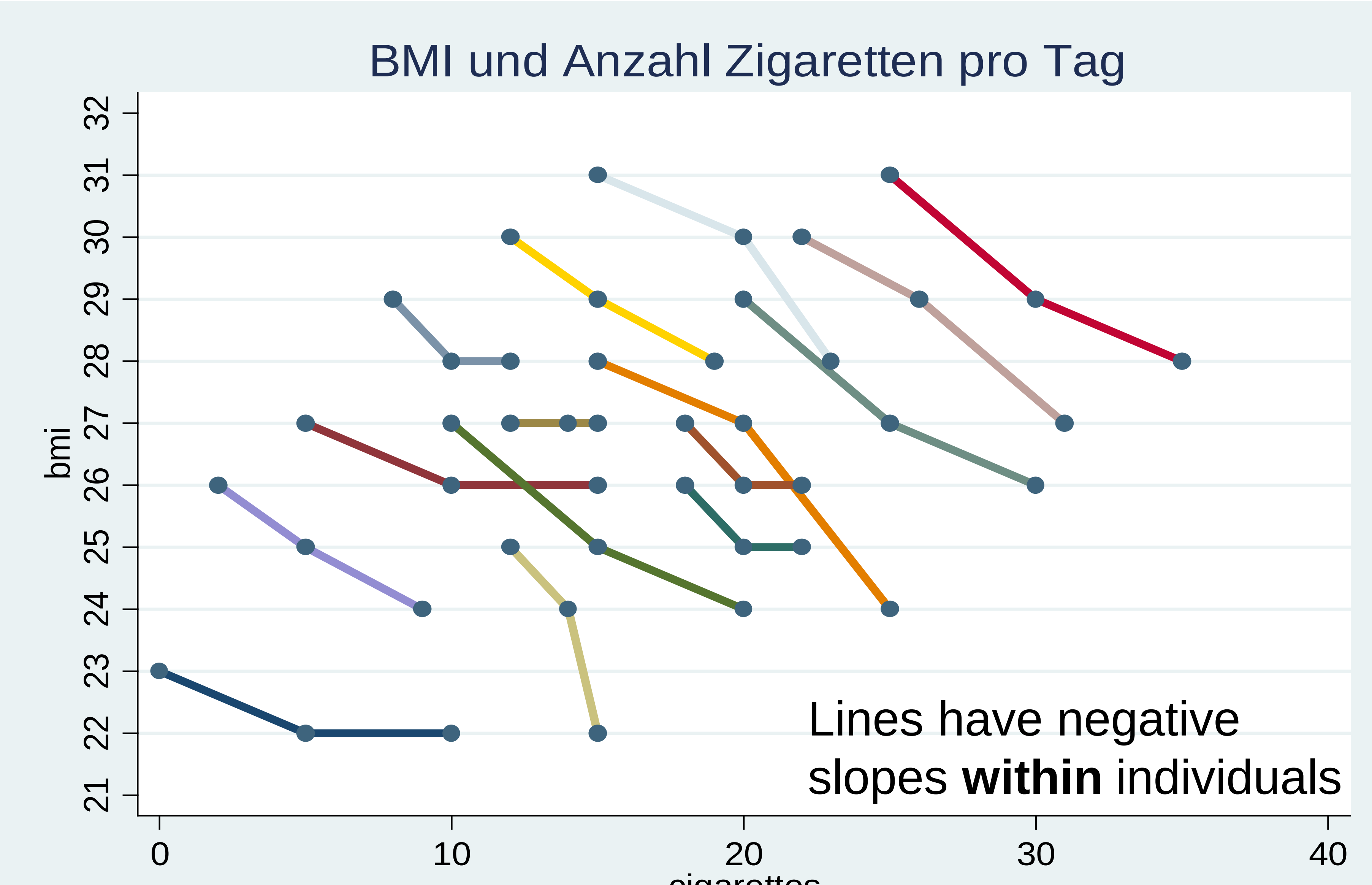
BMI higher by .17 if number of cigarettes is 1 higher

Omitted variable: Social class!

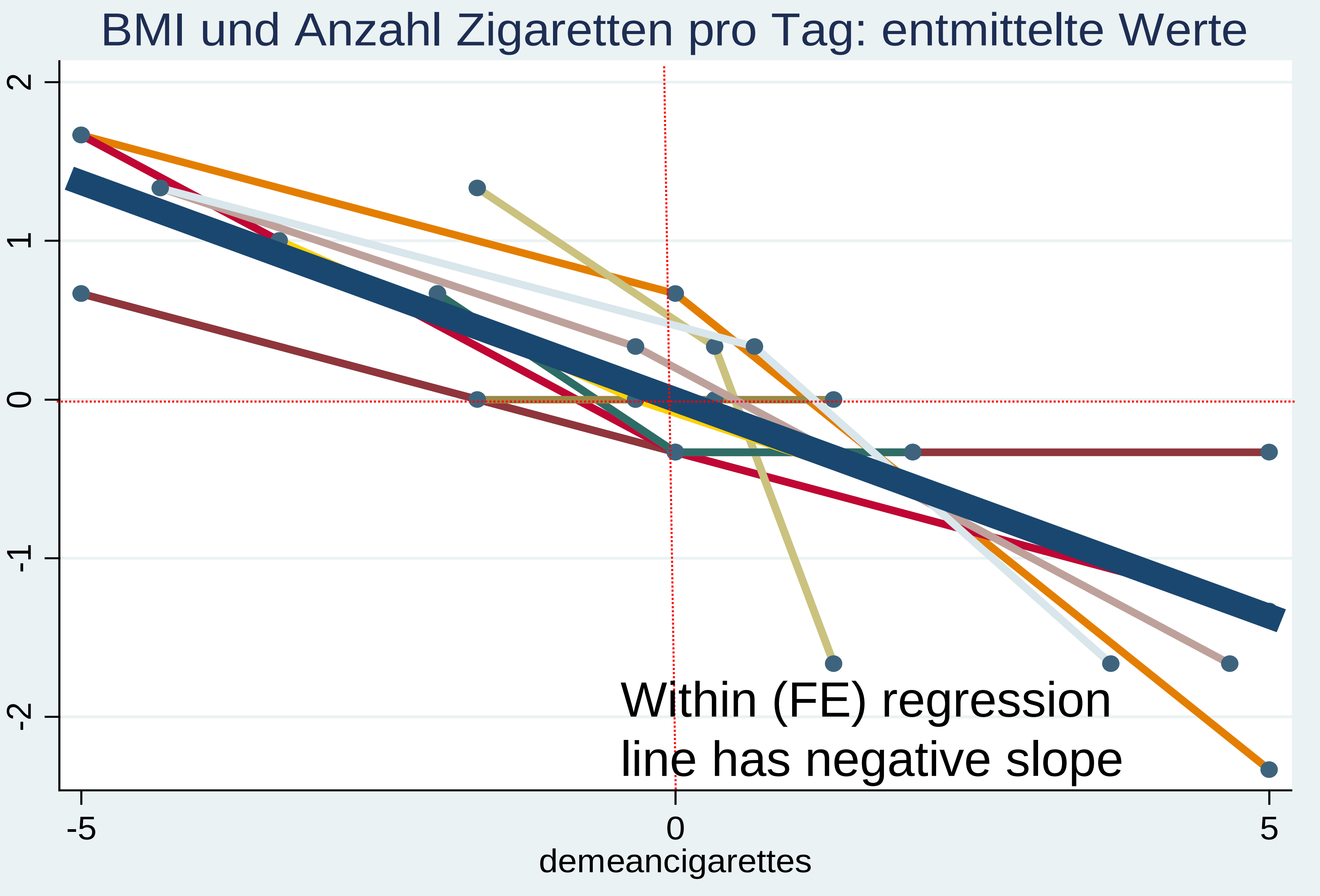


Panel Data is even better:
We can take a look **within individuals**

Panel Data: Observations are clustered in Individuals!



Individually de-meaned values: OLS curves through Origin (0,0)



How can we calculate a within-regression coefficient?

Error components in panel data models

- We separate the error components:

  $e_{it} = \alpha_i + \varepsilon_{it}$, α_i = person-specific **unobserved heterogeneity** (level) = „fixed effects“ (e.g., social origin, ability)

  ε_{it} = „residual“ (e.g., sunshine)

  Model:

$$bmi_{it} = \beta_0 + \beta_1 x_{it} + \alpha_i + \varepsilon_{it}$$

- Remember: Pooled OLS assumes that x is not correlated with both error components α_i and ε_{it} (omitted variable bias)

Fixed effects regression

- We can eliminated the fixed effects α_i by estimating them as *person specific* dummies
- -> remains **only within-variation**

- Corresponds to "de-meaning" for each individual:

$$bmi_{it} = \beta_1 x_{it} + \alpha_i + \varepsilon_{it} \qquad (1)$$

individual mean:

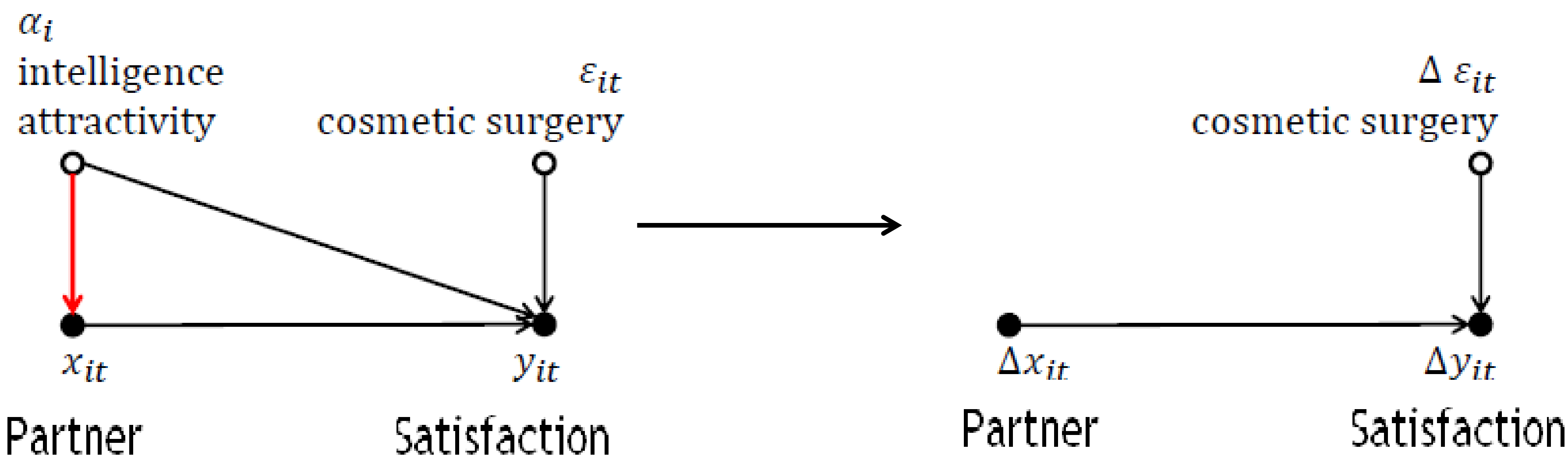
$$\overline{bmi}_i = \beta_1 \bar{x}_i + \alpha_i + \bar{\varepsilon}_i \qquad (2)$$

- subtract (2) from (1):

$$bmi_{it} - \overline{bmi}_i = \beta_1 (x_{it} - \bar{x}_i) + (\varepsilon_{it} - \bar{\varepsilon}_i)$$

-> **Fixed (all time invariant) effects $\boldsymbol{\alpha_i}$ disappear**, i.e. person-constant unobserved heterogeneity is eliminated

Graphical interpretation of de-meaning



Because $\boldsymbol{\alpha_i}$ is not in the differenced equation, $E(\alpha_i \mid x_{it}) = 0$ is no longer required for consistency

De-meaning identifies the causal effect under weaker assumptions

OLS of individually de-meaned Data

We **de-mean and regress** the Data:

```
egen mbmi=mean(bmi), by(id)
egen mcigarettes=mean(cigarettes), by(id)

gen wbmi=bmi-mbmi
gen wcigarettes=cigarettes-mcigarettes

. reg wbmi wcigarettes

      Source |       SS       df       MS              Number of obs =      45
-------------+------------------------------           F(  1,    43) =  147.78
       Model |   34.082846     1   34.082846           Prob > F      =  0.0000
    Residual |    9.917154    43  .230631488           R-squared     =  0.7746
-------------+------------------------------           Adj R-squared =  0.7694
       Total |          44    44           1           Root MSE      =  .48024

------------------------------------------------------------------------------
        wbmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 wcigarettes |  -.2733918   .0224893   -12.16   0.000    -.3187459   -.2280377
       _cons |  -2.86e-07   .0715901    -0.00   1.000    -.1443755    .1443749
```

BMI *de*creases by .27 with each additional cigarette

wrong standard error

Direct modeling of fixed Effects in Stata

xtreg bmi time, fe (calculates correct df; this causes higher Std. Err.)

```
. xtreg bmi cigarettes, fe

Fixed-effects (within) regression               Number of obs      =        45
Group variable: id                              Number of groups   =        15

R-sq:  within  = 0.7746                         Obs per group: min =         3
       between = 0.3495                                        avg =       3.0
       overall = 0.1089                                        max =         3

                                                F(1,29)            =     99.67
corr(u_i, Xb)  = -0.8080                        Prob > F           =    0.0000

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  cigarettes |  -.2733918    .027385    -9.98   0.000    -.3294003   -.2173833
       _cons |    31.1989   .4622757    67.49   0.000     30.25344    32.14436
-------------+----------------------------------------------------------------
     sigma_u |  3.6906387
     sigma_e |  .58478272
         rho |  .97550841   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(14, 29) =    41.48              Prob > F = 0.0000
```

Alternative: OLS with individual dummies controlled

```
. xi i.id, noomit
. reg bmi cigarettes _I* , noconst

      Source |       SS       df       MS              Number of obs =      45
-------------+------------------------------           F( 16,    29) = 5889.41
       Model |  32224.0828    16  2014.00518           Prob > F      =  0.0000
    Residual |    9.917154    29  .341970827           R-squared     =  0.9997
-------------+------------------------------           Adj R-squared =  0.9995
       Total |       32234    45  716.311111           Root MSE      =  .58478

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  cigarettes |  -.2733918    .027385    -9.98   0.000    -.3294003   -.2173833
       _Iid_1 |   23.70029   .3643332    65.05   0.000     22.95515    24.44544
       _Iid_2 |   29.06725   .4347228    66.86   0.000     28.17814    29.95636
       _Iid_3 |   29.43421   .5317197    55.36   0.000     28.34672     30.5217
       _Iid_4 |   31.80117   .6434009    49.43   0.000     30.48527    33.11707
       _Iid_5 |   34.16813   .7633481    44.76   0.000     32.60691    35.72935
       _Iid_6 |   37.53509   .8882188    42.26   0.000     35.71848     39.3517
…
```

useful for small N, the u_i are estimated (only approximate)

Summary: Fixed Effects Estimation

- Solves problem of **time-invariant** unobserved heterogeneity
- Causal interpretation of coefficients

But:

- If number of groups large, **many extra parameters**
- Enough **within-variance needed** in data
- Estimation of ***person-constant* covariates** (like sex) not possible, **dropped from the model.** But: possibility to use **interactions** with time-changing variables (like sex*nrchildren: include main effect nrchildren)
- Measurement errors (change!) may cause problems
- Assumption that most important omitted variables are time-invariant is quite strong

Fixed Effects Model example from literature: Does civic engagement increase generalised trust?

Source: Van Ingen, E., & Bekkers, R. (2013). Generalized Trust Through Civic Engagement? Evidence from Five National Panel Studies. *Political Psychology.*

- Data: Swiss Household Panel 2004-2008
- Variables:
  - Y: Belief that most people can be trusted (scale 0 – 10)
  - X: Number of memberships in voluntary associations (0 - 9)
  - Control: Education, health, employment, having a partner

- Cross-sectional interpretation : compare trust of members/non-members with more or less membership
- Longitudinal interpretation : does trust change once individuals join or quit organisations?

Civic engagement and trust example: between and within regression

	Between	**Fixed Effects**
Membership count	**0.413****	**0.037**
Education	0.089**	0.008
Partner	-0.262**	0.015
Health: not well (ref)		
so, so / average	0.293	0.023
well	1.109**	0.064
very well	1.278**	0.069
Employed	0.005	-0.206**
Intercept	3.676**	5.263**

n= 13’534 observations, 4’436 individuals; Controlled for year of measurement

Source: Van Ingen and Bekkers (2013)

Civic engagement and trust: assumptions / limitations of FE model

- All transitions are assumed to have the **same effect** (general assumption of linear regression)
  - Effects of joining and quitting an organisation are symmetric
  - Effect of maintaining 4 memberships (4-4) and staying uninvolved (0-0) are equal

In addition:

- Other life events may impact both membership and trust (third variable)

Civic engagement and generalised trust: First-difference model excursus

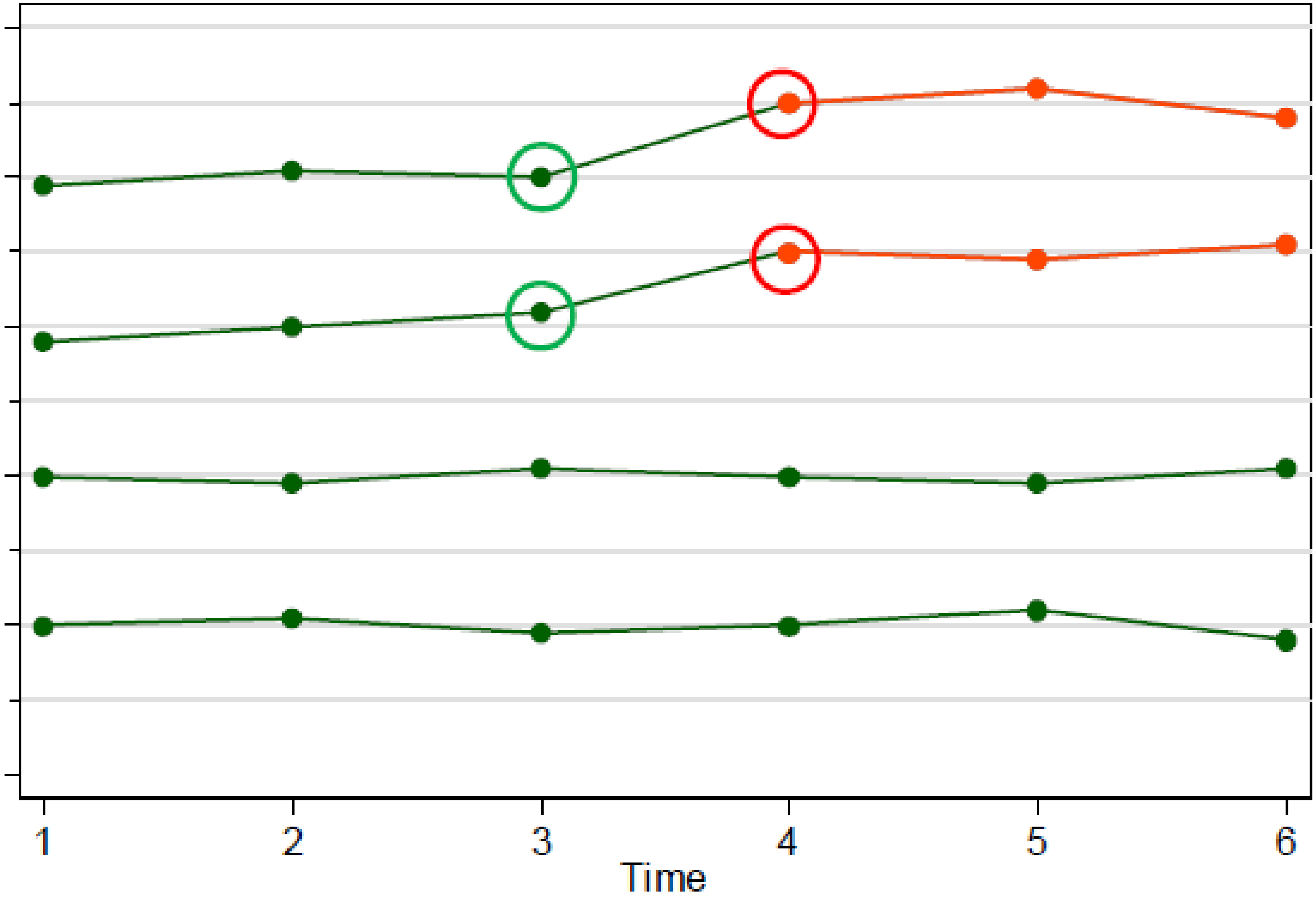
- Distinction between 4 groups to test for asymmetry:
  - Non-participants: remain uninvolved (Reference groups)
  - Join organisation (entry): from 0 to at least 1 membership
  - Quit organisation (exit): from at least 1 membership to 0
  - Participants: remain involved

- Compares change in trust of four groups at two time points
  $y_t - y_{t-1} = \mathrm{a} + b_1 * entry + b_2 * exit + b_3 * stay\ involved + \cdots . + e$

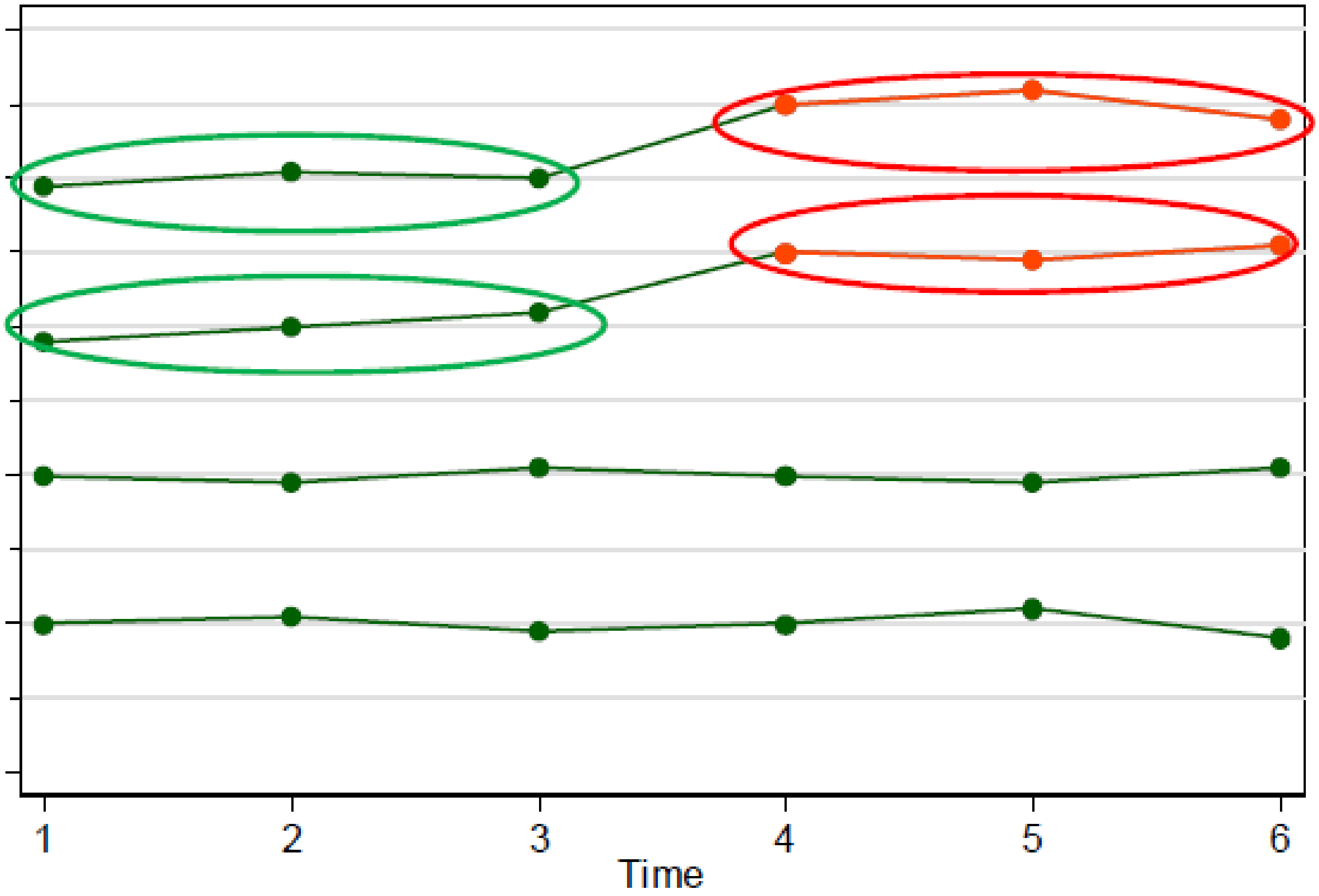
- Advantage: theoretically less restrictive
- Disadvantage: captures only short-term effects

More observations lost due to gaps

Graphical interpretation of First-difference model



Remember: Graphical interpretation of FE model



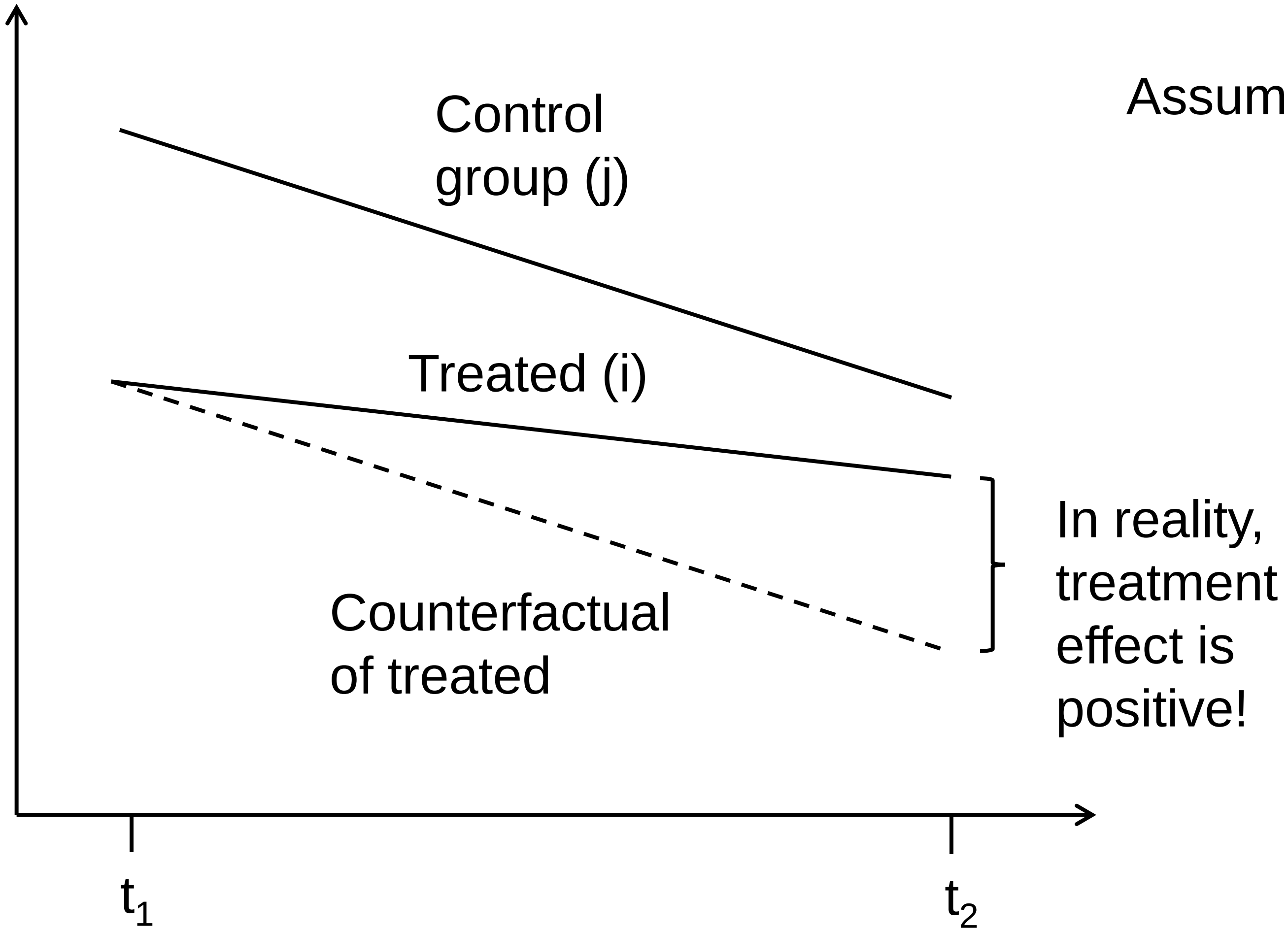
Civic engagement and generalised trust: (FD) results

	Between (n=13'534)	FE (n=13'534)	First difference (n=8327)
Membership count	**0.413****	**0.037**	
Remain uninvolved (n=1485) (ref)			**0**
Entry / start (n=810)			**0.178***
Exit / quit (n=791)			**0.038**
Remain involved (n=5333)			**0.109***
Education	0.089**	0.008	0.007
Partner	-0.262**	0.015	0.009
Health: not well (reference)	0	0	0
so, so / average	0.293	0.023	0.310*
well	1.109**	0.064	0.173
very well	1.278**	0.069	0.138
Employed	0.005	-0.206**	0.133**
Intercept	3.676	5.263**	0.083**

What if we have a treatment which does not account for general developments (e.g., trend)?

With Panel data II: "difference-in-difference" (DID):

$$(Y_{\mathbf{i,t_2}}^{Treat} - Y_{\mathbf{i,t_1}}^{Treat}) - (Y_{\mathbf{j,t_2}}^{NonTreat} - Y_{\mathbf{j,t_1}}^{NonTreat})$$



Assumption: parallel trends

DiD

Comparison of groups at different time points (a version of FE-model)

i.e., we calculate treatment effect and control for time

'DID' – estimator in case of a simple treatment:

$$\underbrace{\textbf{(after}_{\textbf{treat}}\textbf{ - before}_{\textbf{treat}}\textbf{)}}_{\text{FE/within}}\textbf{ - }\underbrace{\textbf{(after}_{\textbf{control}}\textbf{ – before}_{\textbf{control}}\textbf{)}}_{\text{trend}}$$

We can also include time dummies or (linear) trend

Fixed Effects Individual Slopes (FEIS) models

$$y_{it} = \beta_0 + \beta_1 x_{it} + \alpha_{1i} + \alpha_{2i} t + \varepsilon_{it}$$

Individual level and linear slope controlled: only difference around **individual** trend.

-> weaker assumption than standard FE: part of ε_{it} which is due to individual trend (a_{2i}t) needs not be independent of x_{it}

-> In FEIS model time-varying unobserved heterogeneity that is due to **individual-specific trends** is no longer a problem

Use ado xtfeis.ado in Stata

7

Random Effects Models

Motivation: multilevel (RE) models

If data have different levels with

- observations are not independent of levels *and*

- There true social interactions

Examples:

Schools – classes – students: first applications

Networks: people are influenced by their peers

Spatial context: from environment (e.g., poor people are less happy if they live in a rich environment) – US: “neighborhood-effects”

Interviewer - effects: respondents clustered in interviewers

and:

Panel-surveys: waves clustered in respondents (households)

Within versus cross-sectional research questions

“Within”– “**causal**” effects of *time-variant* variables:

→ modeling intrapersonal change (FE models)

Cross-sectional – association with *time-invariant* effects:

→ OLS with robust standard errors

In unbalanced panels:

→ RE models

Interpretation (e.g. presence of children):

within: effect of additional children

between: differences between people with a different number of children

Starting point RE: “null” (“Variance Components” (VC)) model

$y_{it} = \alpha_{0i} + \varepsilon_{it}$ (note : no intercept a in VC model)

where :

α_{0i} = individual specific random variable (N(0, σ_{u0}) assumed); "between"

ε_{it} = deviation from individual specific mean (N(0, σ_{ε}) assumed); "within"

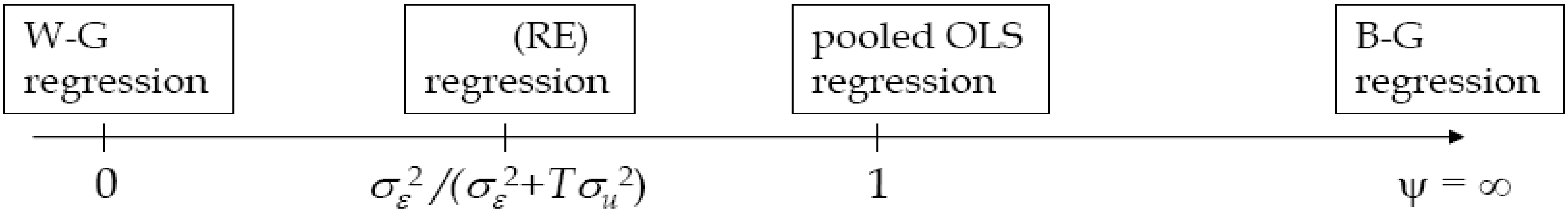
the VC model allows for variance decomposition :

ρ = correlation between different time points t within an individual i :

$$\rho = \frac{\sigma_{u0}^2}{\sigma_{u0}^2 + \sigma_{\varepsilon}^2}$$ (= ICC = intra - class - correlation = autocorrelation in Panels)

(note : ρ significant $\rightarrow$ multilevel model necessary)

Idea RE: weighted within and between



RE - Regression is equivalent to pooled OLS after the Transformation :

$$(y_{it} - \theta\,\bar{y}_i) = \beta_0(1-\theta) + \beta_1(x_{it} - \theta\,\bar{x}_i) + (u_i(1-\theta) + (\varepsilon_{it} - \theta\,\bar{\varepsilon}_i))$$

with $\theta = 1 - \sqrt{\dfrac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + T\sigma_u^2}}, \quad 0 < \theta < 1$

σ_u^2 large (bias from omitted time-invariant variables may cause trouble)

$\rightarrow \theta$ close to 1

$\rightarrow$ RE close to FE

RE allows estimation of time-invariant variables u_i

RE biased because u_i remains in error term (if $\text{cov}(x, u_i) \neq 0$)

Example Θ based on Satisfaction / Partner data

```
. xtreg satlife partner, re theta

Random-effects GLS regression                   Number of obs      =        24
Group variable: id                              Number of groups   =         4

R-sq:                                           Obs per group:
     within  = 0.8982                                         min =         6
     between = 0.8351                                         avg =       6.0
     overall = 0.4065                                         max =         6

                                                Wald chi2(1)       =    128.82
corr(u_i, X)   = 0 (assumed)                    Prob > chi2        =    0.0000
theta          = .9613836

------------------------------------------------------------------------------
     satlife |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     partner |    1.00596   .0886305    11.35   0.000     .8322479    1.179673
       _cons |    4.99851   .8120631     6.16   0.000     3.406896    6.590124
-------------+----------------------------------------------------------------
     sigma_u |  1.4131587
     sigma_e |  .13377114
         rho |  .99111886   (fraction of variance due to u_i)
```

- Estimate (1.006) is close to that from FE model (1.000) because Θ close to 1
- About 90% of variance is explained by partnership status change

Decision if FE or RE appropriate: Hausman test

Test if FE or RE model (**basic assumption: FE unbiased**)
Test H_0: $E(u_i \mid x_{it}) = cov(u_i, x_{it}) = 0$

$cov(u_i, x_{it}) = 0$ -> **FE and RE unbiased**, **FE is inefficient** -> RE
$cov(u_i, x_{it}) \neq 0$ -> **FE is unbiased and RE is biased** -> FE

If H_0 is true (between-coeff.=within-coeff.), no differences between FE and RE

equivalently:

Hausman compares estimation coefficients $\hat{\beta}_{FE}$ and $\hat{\beta}_{RE}$

if H_0, $\hat{\beta}_{FE} = \hat{\beta}_{RE}$ and $\hat{\beta}_{RE}$ more efficient $(var(\hat{\beta}_{FE}) > var(\hat{\beta}_{RE}))$

if H_1, $\hat{\beta}_{FE}$ unbiased but $\hat{\beta}_{RE}$ not

Note:
- H_0 almost always rejected (sample size high enough even with small differences)
- **Test is only formal and does *not* replace research question driven check for model appropriateness**

Hausman test: example

- **Hausman Test: RE or FE estimate?**

```
xtreg satlife partner, re
estimates store randeff
xtreg satlife partner, fe
estimates store fixdeff
hausman fixdeff randeff, sigmamore
```

```
 ---- Coefficients ----
            |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
            |    fixdeff      randeff       Difference          S.E.
------------+----------------------------------------------------------------
     partner |    .9999999      1.00596       -.0059605        .0024201
-----------------------------------------------------------------------------
                           b = consistent under Ho and Ha; obtained from xtreg
            B = inconsistent under Ha, efficient under Ho; obtained from xtreg

    Test:  Ho:  difference in coefficients not systematic

                  chi2(1) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =        6.07
                Prob>chi2 =      0.0138
```

FE versus RE models

Regression equation: $y_{it} = \alpha + \beta x_{it} + u_i + \varepsilon_{it}$

Fixed effects models	**Random effect models**
• OLS estimated • only variance **within-** individuals used • Controls for **unobserved heterogeneity (consistent also if Cov(u,x)≠0)** • Effects of **time-invariant characteristics cannot be estimated** (e.g., gender, cohort) → If research interest is **longitudinal or causal**	• Cannot be estimated with OLS • Uses both within- and between- individuals variance • assumes **exogeneity: Cov(u,x)=0 (no effects from unobserved variables)** • Effects from **time-invariant and time-varying** covariates → If research interest is **1) cross-sectional** or **2) on variance on different levels**

- formal test for RE against FE model: Hausman test (test of unbiasedness)

FE versus RE models: substantive questions

- Within estimators cannot estimate the effects of time-constant variables
  – sex, nationality, social origin, birth cohort, etc.

-> panel data do not help to identify the causal effect of time-constant variables

-> the "within logic" applies only with time-varying variables (Something must "happen")

Only then a before-after comparison is possible: Analyzing the effects of events

- Such questions are the main strength of panel data and the within methodology
  – [Event variables can not only be categorical, but also metric]

- If one has substantive interest in the effect of a time constant regressor, one could estimate group-specific FE models (e.g., for men and women separately).

Fixed und random effect example, Hybrid model

DepVar: Wage	Model 1	Model 2		Model 3		Model 4		Model 5
(Z-values)	FE	RE	RE-FE	RE	RE-FE	RE	RE-FE	(Hybrid Model)
Occupational status (SEI/10)	**.037 (6.40)**	.046 (8.67)	.009 (4.06)	.046 (8.56)	.009 (4.06)	.039 (7.11)	.002 (0.79)	.037 (6.40)
Union	**.083 (3.93)**	.121 (6.22)	.038 (4.95)	.124 (6.39)	.041 (4.95)	.124 (6.41)	.041 (4.72)	.083 (3.93)
Schooling (years) (time invariant)						0.64 (7.30)		.056 (5.62)
Black (time invariant)				-.140 (2.89)		-.130 (2.77)		-.150 (3.17)
SEI time mean (time invariant)								.029 (1.71)
Union time mean (time invariant)								.029 (1.71)
Hausman chi-square			45.3		46.8		24.6	

Vella & Verbeek 1998, Wooldridge 2003, Halaby 2007

8

Non linear regression

Non-linear regression: motivation

- Linear regression: requires continuous dependent variable

  e.g. BMI, income, satisfaction on scale from 0-10 (?)
- Most variables in social science are not continuous but discrete
  - Opinions: agree vs. disagree
  - Poverty status
  - Party voted for
  - Number of visits to the doctor
  - Having a partner
- We need appropriate regression models!

Non-linear models

Dependent variable is not continuous: non-linear regression

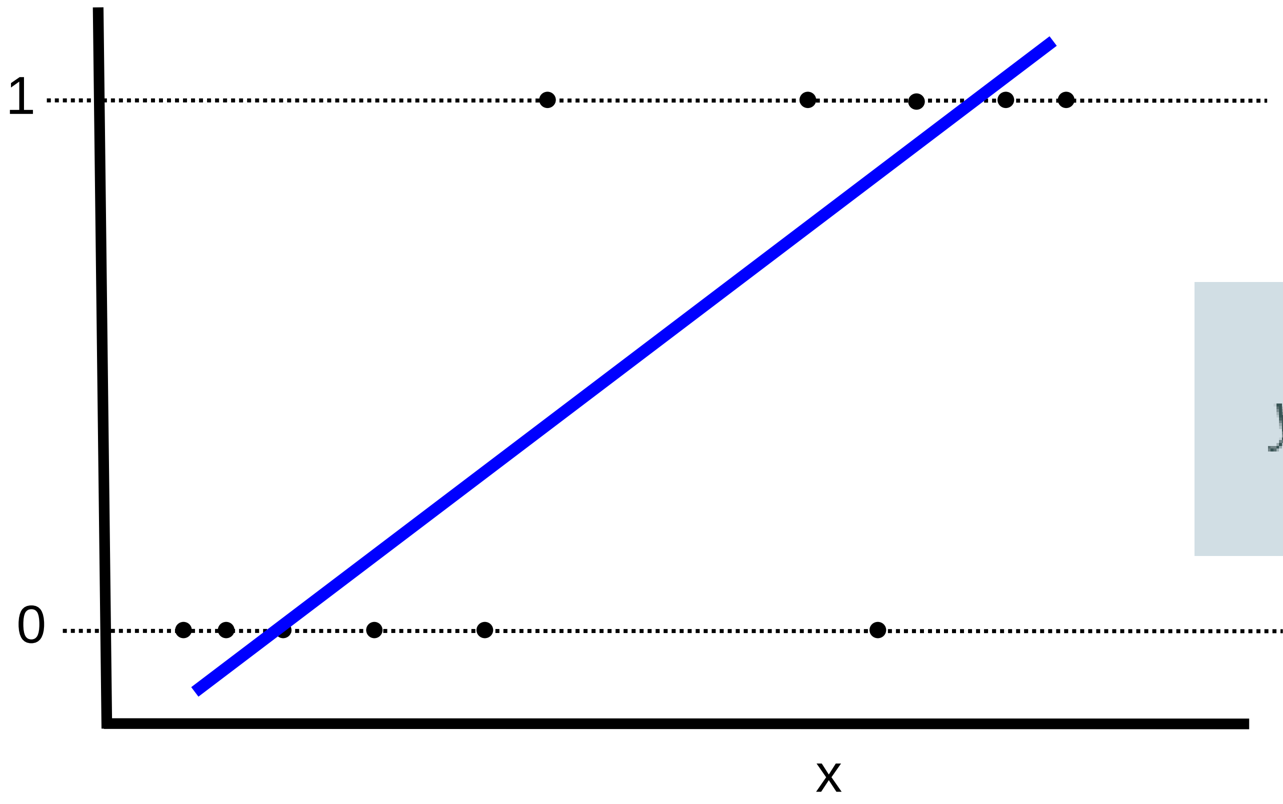
Binary variables (dummy variables, 0 or 1) (e.g. yes-no, event – no event)	Logistic Regression, Probit Regression, and many more
Multinomial (unordered variables) (e.g. vote choice, occupation)	Multinomal logistic Regression Multinomial probit Regression
Ordinal (e.g. satisfaction)	Ordinal Regression
Count variable (e.g. doctor visits)	Poisson Regression Negative Binomial Regression

Linear probability model for binary variables

$E(Y) = P(Y_i = 1)$

Linear probability model: $P(Y_i = 1 \mid X_i) = \alpha + \beta_1 x_1 + \ldots.$

Estimation with OLS regression



$$y_i = \begin{cases} 1 & \text{if } y_i^* \geq 0 \\ 0 & \text{if } y_i^* < 0 \end{cases}$$

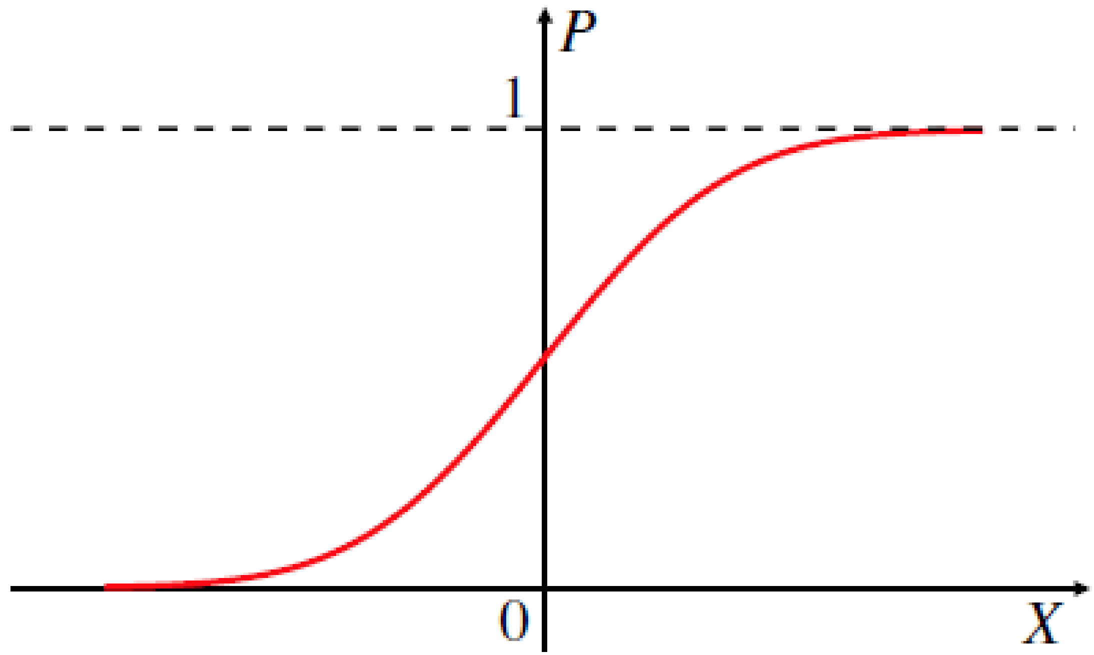
Advantages and problems of linear probability model

- Advantages
  - Estimation with OLS regression
  - Direct interpretation of coefficients
  - Less biased if P(Y=1) not too close to 0 or 1
- Problems: violation of regression assumptions
  - Predicted probabilities may be negative or greater than one
  - Relationship between response probability and x may not be linear, especially for P(Y=1) close to 0 and P(Y=1) close to 1
  - The variance of y for binary variables is P(Y=1) * P(Y=0)
    → residual variance depends on x
    → heteroskedasticity
  - Residuals can take only two values for fixed x
    → residuals are not normally distributed

S-shaped function

- P(Y=1) between 0 and 1
- Non-linear
- Symmetry
- Often used function:
  Cumulative logistic distribution
  (Logit model)

$$\Pr(y=1)=\frac{1}{1+e^{-(\alpha+\beta_1 x_1+\beta_2 x_2+\ldots)}}$$



- Other functions also used (e.g. probit). For practical purposes, these models provide very similar predicted probabilities

Generalised linear model

- A latent (unobserved) continuous variable y* which underlies the observed data

$y^* = a + b_1x_1 + \ldots + e^*$

$$y_i = \begin{cases} 1 & \text{if } y_i^* \geq 0 \\ 0 & \text{if } y_i^* < 0 \end{cases}$$

- Assume y_i^* is generated by a linear regression structure
- Link function between y and y*: $E(y) = f(y^*) = f(a + b_1x_1 + \ldots + e^*)$

  e.g. logit, probit, poisson, negative binomial, identity
- Because y* is not observed
  - e_i^* are not observed, variance of e_i^* has to be assumed

    Logit model: e_i^* standard logistic (with var = $\pi^2/3 \sim 3.29$)

Maximum likelihood estimation (MLE)

- Usually, non-linear models are estimated by maximum likelihood
- Principle for MLE: Which set of parameters has the highest likelihood to generate the data actually observed (x_i, y_i)?
- Advantages
  - Extremely flexible and can easily handle both linear and non-linear models (Linear model: MLE = OLS estimator)
  - Desirable asymptotic properties: consistency, efficiency, normality, (consistent if missing at random MAR)
- Disadvantages
  - Requires assumptions on distribution of residuals
  - Desired properties hold only if model correctly specified
  - Best suited for large samples
- Often, there is no closed form (algebraic) solution. Coefficients have to be estimated through iteration methods

Example: logistic regression

```
.logit svp female age1830 age4660 age60plus satdem

Logistic regression      Number of obs      =      6,224
        LR chi2(5)          =      124.97
        Prob > chi2         =      0.0000

Log likelihood = -2193.4524     Pseudo R2         =      0.0277
```

svp	Coef.	Std. Err.	z	P>z	[95% Conf.	Interval
female	**-.4469116**	**.0798622**	**-5.60**	**0.000**	**-.6034387**	**-.2903845**
age1830	**-.0496199**	**.1343308**	**-0.37**	**0.712**	**-.3129034**	**.2136636**
age4660	**-.0781095**	**.1185982**	**-0.66**	**0.510**	**-.3105577**	**.1543388**
age60plus	**.0865025**	**.1152573**	**0.75**	**0.453**	**-.1393976**	**.3124026**
satdem	**-.1987384**	**.0199542**	**-9.96**	**0.000**	**-.2378478**	**-.1596289**
_cons	-.5746696	.1567911	-3.67	0.000	-.8819746	-.2673647

Interpretation of non linear models

- y^* has no units, scale of y^* changes if additional x_i are included
- Because of the non-linearity, effects depend on values of x and cannot be interpreted directly ($\rightarrow$ not constant)
- Coefficients cannot be compared across different models
- Interpretation of coefficients
  - Qualitative interpretations (direction and significance level)
  - Odds ratio (problematic)
  - Predicted probabilities

Excursus: Odds ratios (OR)

OR often misunderstood as relative risk

		P	Odds $\left(\frac{P}{1-P}\right)$	OR $\left(\frac{Odds\ Group 1}{Odds\ Group\ 2}\right)$	RR $\left(\frac{P\ Group\ 1}{P\ Group\ 2}\right)$
A	Group 1	0.10	0.11	2.10	2.00
	Group 2	0.05	0.05		
B	Group 1	0.40	0.67	2.70	2.00
	Group 2	0.20	0.25		
C	Group 1	0.80	4.00	6.00	2.00
	Group 2	0.40	0.67		
D	Group 1	0.60	1.50	6.00	3.00
	Group 2	0.20	0.25		
E	Group 1	0.40	0.67	6.00	4.00
	Group 2	0.10	0.11		

Ref.: Best and Wolf 2012, Kölner Zeitschrift für Soziologie

Compute predicted probabilities

- Remember: predicted probabilities depend on values of x and parameter estimates (and unobserved heterogeneity)
- Predicted probabilities are estimates
  → confidence intervals
- Discrete change: predict probabilities for different values of x
- Marginal effect or partial effect: The slope of Pr(y=1) at x.
- Two methods
  - Adjusted predictions: Specify values for each of the independent variables, compute probability for individual who has those values
    Usually: x at the mean; Alternative: representative values
  - Average effects: Compute predicted probability for each individual at observed values of x. Average probability over all individuals (average marginal effect, average adjusted predictions)

Predicted probabilities: example

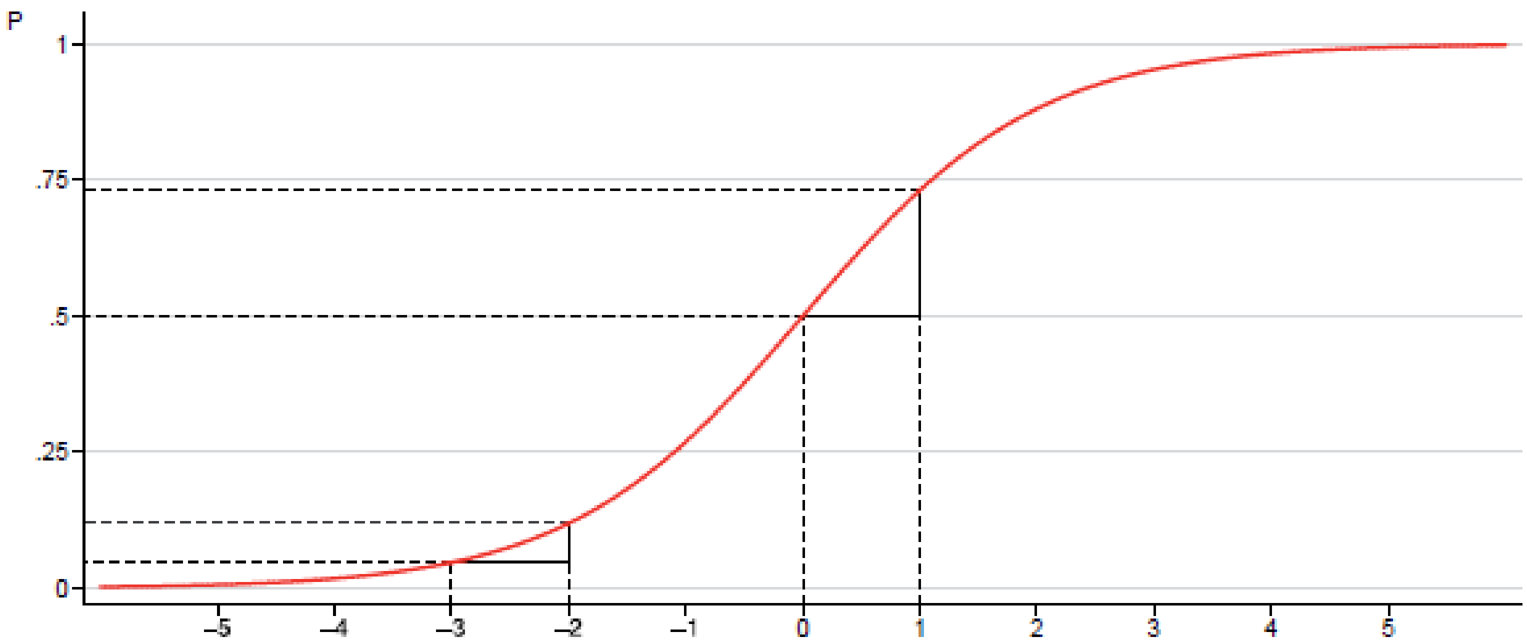
Logit model
$$\Pr(y=1) = \frac{1}{1+e^{-(\alpha+\beta_1 x_1+\beta_2 x_2)}} = \frac{\exp(\text{logit})}{1+\exp(\text{logit})}$$

Hypothetical regression result: y*=3 - 2*x_1+ 0.5* x_2

Example Individual i with x_1=2 and x_2=1,

$$P(Y_i=1|X_i)$$
$$= \frac{1}{1+e^{-(3+(-2)2+0.5)}}$$
$$= \frac{1}{1+e^{0.5}} = 0.62$$

P(Y=1)



X

Example: Average adjusted predictions (AAP)

```
logit svp i.female i.agegr satdem
margins female agegr
```

	Margin	Delta-method Std. Err.	z	P>z	[95% Conf.Interval]	
female						
0	.1434225	.0066169	21.68	0.000	.1304537	.1563914
1	.0974338	.0049952	19.51	0.000	.0876435	.1072242
agegr						
18-30	.1131354	.0094684	11.95	0.000	.0945777	.131693
31 - 45	.1180999	.0095385	12.38	0.000	.0994048	.136795
46 - 60	.1103665	.0069987	15.77	0.000	.0966493	.1240837
over 60	.127197	.0072908	17.45	0.000	.1129074	.1414866

Example: continuous variable (satisfaction democracy)

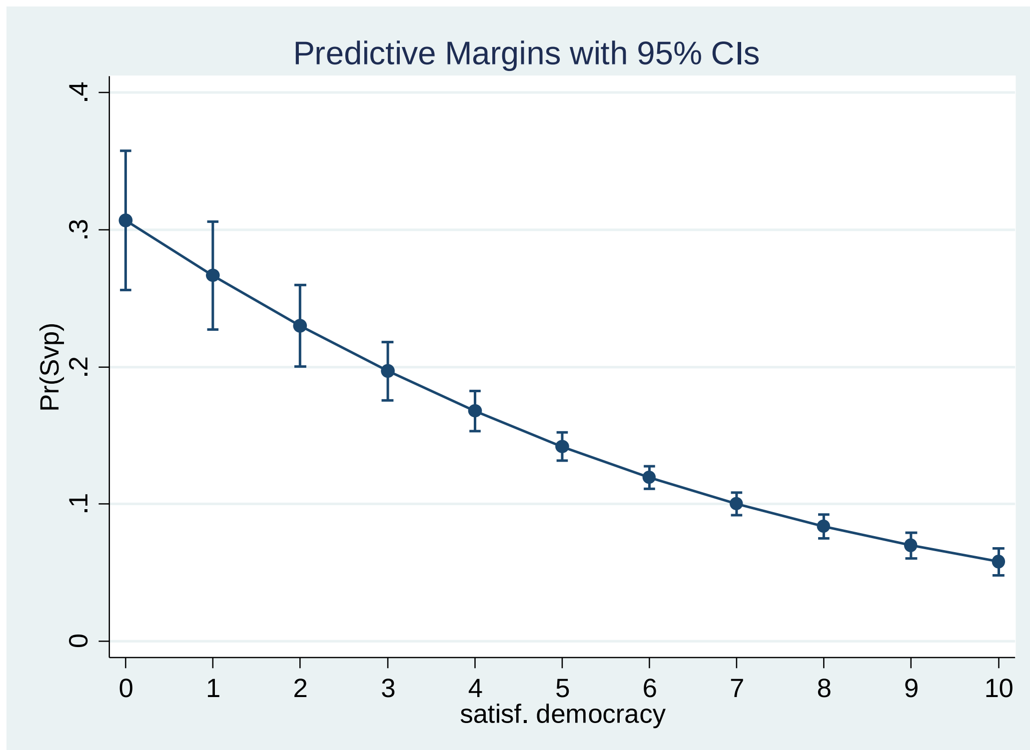
1. Average marginal effects

```
margins , dydx(satdem)
```

dy/dx	Std. Err.	Z	P>z	[95% Conf.	Interval
satdem	-.0201908	.0020354	-9.92	0.000	-.0241801	-.0162015

2. Average adjusted predictions

Average adjusted predictions



Model performance

- Linear regression: R^{2}, adjusted R^2

$$R^2 = \frac{\text{explained var. in y}}{\text{total var. in y}}$$

- Non linear regression
  - variance of the residual not observed
  - many so-called pseudo-R^2 (0,1) (Stata: fitstat)

```
Log-Lik Intercept Only:    -2255.939  Log-Lik Full Model:   -2193.452
D(6216):                    4386.905  LR(5):                124.973
                                      Prob > LR:            0.000
McFadden's R2:                 0.028  McFadden's Adj R2:    0.024
Maximum Likelihood R2:         0.020  Cragg & Uhler's R2:   0.039
McKelvey and Zavoina's R2:     0.052  Efron's R2:           0.020
Variance of y*:                3.469  Variance of error:    3.290
Count R2:                      0.882  Adj Count R2:         0.000
AIC:                           0.707  AIC*n:                4402.905
BIC:                      -49917.116  BIC':                 -81.292
```

The likelihood ratio test

- To test hypotheses involving several predictors (multiple constraints) (e.g. Test $\beta_2 = \beta_3 = 0$)
- Compare log-likelihoods of constrained and unconstrained model, e.g.
  - M_u: $\pi=(F(\alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3)$
  - Mc= $\pi=(F(\alpha + \beta_1 x_1)$
- Generally: $L_c \leq L_u$
  - Constraints valid: $L_u - L_c = 0$
  - Constraints invalid: $L_u - L_c > 0$
- Test statistic: $LR = 2(L_u - L_c) \sim X^2(q)$; (q: number of constraints, d.o.f.)
- Prerequisites of LR test
  - Models are based on the same sample
  - Models are nested

LR test example: test for joint significance

- Vote intention model (support SVP vs. supporting another party
- $P(Y = 1|X) = F(\alpha + \beta_1(\text{female}) + \beta_2(\text{age})) + \beta_3(\text{lnincome}) + \beta_4(\text{contra EU}) + \beta_5(\text{for nuclear energy}) + \beta_6(\text{satisfaction democracy})$
- Do demographic variables (age, sex) matter? H_0: $(\beta_1,\beta_2)=0$

	Unconstrained	Constrained
Female	0.143*	
Age	0.002	
Income	-0.296***	-0.301***
Contra EU	0.834***	0.819***
Pro nuclear energy	0.241***	0.185**
Satisfaction democracy	-0.213***	-0.216***
Log likelihood	-3762.1	-3772.2

- $LR = 2(L_u - L_c) \sim= 2*(-3762.1 - (-3772.2)) = 20.08$
- Chi-squared distribution with 2 degrees of freedom: p=0.0000
  -> we reject H_0 (demographic variables seem to affect the probability to vote for SVP)

Difficulties of nonlinear models: frequent mistakes

- Interpretation of coefficients (Logits, OR)
- Comparison of estimates across models and samples (estimates reflect also unobserved heterogeneity)
  - Be cautious with interpretation
  - Use different measures to show effects (predicted probabilities)
  - Correction proposed by Karlson et al. (2012)

    References: Mood 2010, Best and Wolf 2012, Karlson et al. 2012,
    Stata: ado *khb*
- Interaction effect: cannot be interpreted as in linear models

    References: Ai and Norton 2003, Norton Wan and Ai 2004,
    Stata: ado *inteff*
- Model performance

Multinomial dependent variables

- More than two response categories (m categories)
- Unordered → Multinomial regression
  e.g. Voting preference (different parties), type of education, compare each pair of response categories
  - estimate probability for each category,
    (1 reference category, m-1 equations)
- Ordered → Ordinal regression
  e.g. Opinions (strongly agree, agree, neither , disagree, strongly disagree), health status
  - latent variable with m-1 thresholds
  - estimate cumulative probability (prob. $y \leq mi$)
    (one equation with dummies for m-1 thresholds)

Example: multinomial regression

Voting: FDP/CVP, SP/Greens, other parties; Base category: vote SVP

	FDP & CVP	SP & Greens	Other party	No party
Female	0.377***	0.406***	0.274*	0.583***
Age 18 - 30	-0.162	0.070	-0.167	-0.220
Age 46 - 60	-0.092	0.031	-0.129	-0.075
Age 60+	-0.052	-0.413**	-0.584**	-0.274*
Education: intermed	0.179	0.300*	0.206	-0.135
Education: high	0.780***	0.982***	1.114***	0.287
Income (ln)	0.317***	0.229**	0.397***	0.113
Against EU-integration	-1.704***	-2.790***	-1.559***	-1.665***
Against Foreigners	-0.746***	-1.608***	-1.254***	-0.862***
Pro nuclear energy	-0.064	-1.542***	-0.967***	-0.570***
Satisfaction democracy	0.252***	0.193***	0.183***	0.044
_cons	-3.138***	-0.694	-4.259***	1.364

Refresher: Panel data models in linear regression

Fixed effects models

$Y_{it} = \beta_1 x_i + \beta_1 x_i + \alpha_i + \varepsilon_{it}$

- $\alpha_{i:}$ unobservable stable individual characteristics (as variable, not residual)
- only variance within individuals taken into account
- Control for unobserved heterogeneity (consistent also if $Cov(\alpha,x) \neq 0$)
  -> causal interpretation
- Effects of time-invariant characteristics cannot be estimated (e.g., sex, cohorts)

Random effect models

$Y_{it} = \alpha + \beta_1 x_i + \beta_1 x_i + \alpha_i + \varepsilon_{it}$

- assumes $\alpha_i \sim N(0,\sigma_\alpha)$
- α_i: unobservable stable individual characteristics, part of residual
- Multilevel model with random intercepts
- Controls for unobserved heterogeneity (but consistent only if $Cov(\alpha,x)=0$)
- Effects of time-invariant and time-varying covariates

Fixed effects for non-linear models

- Linear model: by differencing out (or including dummy variables), the u_i disappears from (FE) equation
- Non-linear model: there is no equivalent FE model
  - Incidental parameter problem -> inconsistent estimates
- Instead: Conditional ML estimation (similar to FE)
  - Technical trick to eliminate individual-specific intercepts (number of 1 for each individual as sufficient condition)
  - (Also called Chamberlain fixed-effects model)
  - Only possible for logit and poisson

(here possibly: Logistic Fixed Effects Estimation for two time periods.doc)

- Drawback: Only subsample of individuals with change in y_{it}
  -> information loss
  -> potential bias from excluding stable individuals (external validity)
- Linear probability model (FE) used as alternative

Excursus: Ordinal regression fixed effects

- Cross-sectional analysis: ordered logistic estimation, ordered probit model
- No Fixed Effects estimator, but different Strategies proposed
  - Dichotomise variables and estimate fixed effects logit (choose one cut point)
  - Estimate logistic model with every possible dichotomizing cutoff point and then combine the results (Das and van Soest 1999)
  - Estimate logistic model with every possible dichotomizing jointly (Beatschmann, Staub and Winkelmann 2011)
  - Dichotomise every individual separately (Ferrer-i-Carbonell and Frijters 2004), most frequently at the mean

Random effects for non-linear models

RE model equivalent to linear regression

$y_{it} = F(\alpha_0 + \alpha_i + \mathbf{x}_{1it}\beta_1 + \mathbf{x}_{2it}\beta_2 + \mathbf{...} + \alpha_i + \varepsilon_{it})$ with $\alpha_i \sim N(0, \sigma_\alpha)$, $Cov(\alpha_i, x_i) = 0$

But in contrast to linear models

- Predicted probabilities depend on values of u_i: we have to assume a value for u_i to estimate predicted probabilities
- Measures for variance decomposition questionable
  - only variance of unobserved heterogeneity estimated, within variance is fixed (usually at 1)

  → (ρ)not meaningful
  - Alternative: How much can the unexplained variance between individuals be reduced relative to the empty model?

Stata commands for non-linear panel models

Stata built-in commands

- Random intercept models: xt prefix
  xtlogit, xtprobit, xtpoisson, xttobit, xtcloglog, xtnbreg, xtologit
- Random slope models: meqrlogit, meqrpoisson

Other software necessary for multinomial panel models (run from Stata)

- gllamm add-on (Rabe-Hesketh and Skrondal) to Stata
  (very powerful, freely available (but Stata necessary), slow, become familiar with syntax)
- runmlwin: command to run mlwin software from within Stata (Mlwin software needs to be purchased)

svp	logit	random	fe
	b	b	b
female	0.116**	0.214***	.o
18 - 30 years	**-0.133****	**-0.033**	**0.620*****
46 - 60 years	-0.119**	-0.162**	-0.306***
over 60 years	**-0.097***	**-0.250*****	**-0.601*****
med education	-0.229***	-0.497***	-0.318*
high education	**-0.633*****	**-1.350*****	**-0.537*****
hh income, log	-0.128***	-0.159***	-0.013
stay outside EU	**0.749*****	**0.622*****	**0.049**
prefer Swiss	0.471***	0.402***	-0.010
nuc_pro	**0.225*****	**0.076**	**-0.073**
satisf. democracy	-0.160***	-0.153***	-0.043***
_cons	**1.190*****	**1.818*****	
Lnsig2u _cons		2.034***	
N	55177	55177	26420

RE logistic model for event history analysis

- Example for discrete event history analysis
- Dependent variable
  - 0 event has not occurred
  - 1 event has occurred (since last observation)
- Independent variable
  - Time until event occurrence
  - Any other variable
- Estimate logistic model or random effect logistic model
- Example: change of vote intention (between parties)

Ind.	Wave	Vote intention, Party	Change between parties	Age
1	1	.	.	23
1	2	A	.	24
1	3	B	1	25
1	4	B	0	26
1	5	A	1	27
1	6	No party	0	28

```
Stata: xtlogit change age agesq
```

Example: discrete event history analysis